Discrete-Time Modeling and Synthesis of Musical Instruments

Matti Karjalainen

Helsinki University of Technology
Laboratory of Acoustics and Audio Signal Processing

http://www.acoustics.hut.fi

TEKNILLINEN KORKEAKOULU
TEKNISKA HÖGSKOLAN
HELSINKI UNIVERSITY OF TECHNOLOGY

Content of presentation

- Introduction and motivation
  - Discrete-time and -space modeling
- Physics-based modeling paradigms
  - Digital waveguides
  - Wave digital filters
  - Finite difference models
  - Modal decomposition
  - Source-filter modeling
- Interrelations and mixed modeling
  - K- vs. W-modeling (Kirchhoff vs. wave quantities)
  - Mixed K- and W-modeling
- Real-time simulation and sound synthesis
- Modeling examples + DEMOS!

Discrete-time and -space modeling

- Solving of system equations (frequency domain)
  - Directly solving PDAs
  - Finite Element Method (FEM)
  - Boundary Element Method (BEM)
  - Statistical Energy Analysis (SEA)
- Time-domain simulation and synthesis
  - Digital waveguides (DWG)
  - Wave digital filters (WDF)
  - Finite difference models (DFTD)
  - Modal decomposition
  - Source-filter modeling
- Geometrical approximation of acoustic spaces
  - Ray tracing techniques
  - Image source method
  - Reverberation algorithms

Why using time-domain methods ?

- Computational efficiency
  - Real-time synthesis is not possible through numerical solving of system equations in the frequency domain
  - Digital signal processing (DSP) algorithms available for maximal efficiency
- Modeling of nonlinearities
  - Simulation of nonlinear (non-LTI) systems difficult or impossible in the frequency comain
- Localized element-based computation
  - Representation by elements and their interconnections
- Visualization of the temporal behavior
  - Simulation can be directly visualized
- Embedding in applications
  - Direct applicability in computer music, virtual acoustics, and multimedia applications

What is difficult in discrete time and space ?

- **Time and space resolution problem**
  - **Sampling rate (interval) limits time resolution**
  - **Spatial discretization limits spatial resolution**
- **Causality problem in time-domain simulation**
  - **Unit delay (sample interval) is the shortest period where two-way physical interaction can be simulated explicitly**
  - **Delay-free loop (implicit equation) problem easily encountered**
  - **Nonlinearities particularly pose delay-free loop problems**
- **Nonlinearities introduce the aliasing problem**
  - **Frequency domain folding of signal components -> Signals are easily distorted in an uncontrolled way**

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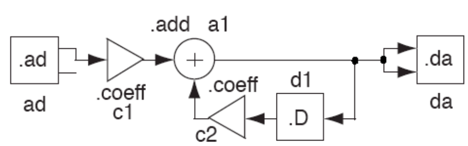
Real-time simulator: BlockCompiler

- **An object-based (block-based) modeling and simulation tool is developed for physical modeling with, e.g.,**
  - ***Physical and DSP blocks are interconnected to patches***
  - ***Macro blocks and multirate computation supported***
  - ***Implemented in Lisp. Blocks and patches generate C code to compile to an executable for real-time simulation***
- **A DSP example:**




```
(patch ((a (.add)))
   (-> (.ad) (.coeff 0.0666) a (inputs (.da)))
   (-> a (.D) (.coeff -0.8668) (input a 1)))
```

K- vs. W-modeling

(Kirchhoff vs. wave quantities)

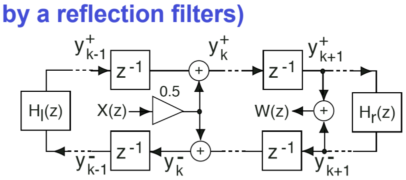
- **Kirchhoff quantities (K-modeling)**
  - Physical quantities that are directly observable in a point
  - Come in the form of dual variables:
    - Force/velocity, pressure/volume velocity, voltage/current
  - or by a single variable plus a parameter:
    - e.g., voltage plus impedance/admittance
  - There is no inherent causality between dual K-variables!
- **Wave quantities (W-modeling)**
  - Waves are easily observable in some physical systems but not in lumped element systems where dimensions are much less than a wavelength
  - Any physical quantity can be split to wave quantities
  - There is a causal relation between the incoming and out-going wave components in a wave port

Digital Waveguide modeling (1)

- Discrete-time modeling of wave propagation and scattering using wave variables (W-modeling):
- Based on the d'Alembert solution of the wave equation in a discretized form, in 1-D case left- and right-traveling waves

$$y(n,m) = \overrightarrow{y}(n-m) + \overleftarrow{y}(n+m)$$
$$\overrightarrow{y}_{k,n+1} = \overrightarrow{y}_{k-1,n} \quad \text{and} \quad \overleftarrow{y}_{k,n+1} = \overleftarrow{y}_{k+1,n}$$

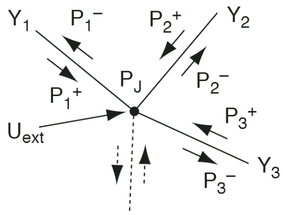
- Wave propagation modelled as a delay-line (sequence of unit delays)
- *Example*: Model of wave propagation in a string or a tube, signal insertion and output, and termination (boundary reflection by a reflection filters)



- Losses and dispersion easily added to the model

Digital Waveguide modeling (2)

- **Wave scattering in a junction of acoustic elements (tubes)**
- **Continuity laws:**
  - ***common pressure P_J and sum of volume velocities = 0***



$$P_1 = P_2 = \ldots = P_N = P_J$$
$$U_1 + U_2 + \ldots + U_N + U_{\text{ext}} = 0$$

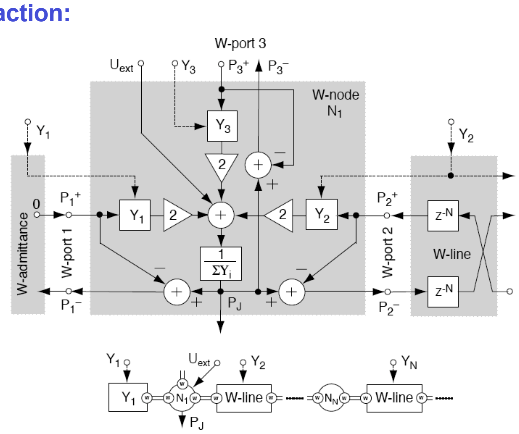
$$P_i = P_i^+ + P_i^- \quad \text{and} \quad U_i^+ = Y_i P_i^+$$

$$P_J = \frac{1}{Y_{\text{tot}}} (U_{\text{ext}} + 2 \sum_{i=0}^{N-1} Y_i P_i^+)$$

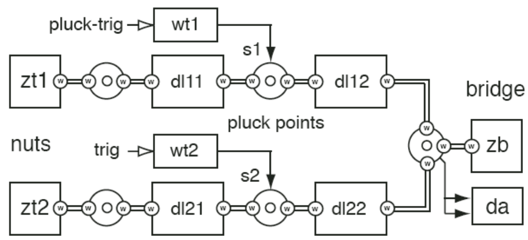
- **Kirchhoff quantities P (pressure) and U (volume velocity)**
- **Wave quantities:**
  - **pressure waves P^+, P^-, P_J**
  - **volume velocity waves U^+, U^-, U_{ext}**
- **Acoustics admittances Y (= volume velocity / pressure)**

Digital Waveguide modeling (3)

- **Scattering junction, passive termination, and delay-line element formulated as a DSP structure and its block-based abstraction:**



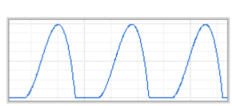
DWG example (1): Two coupled strings

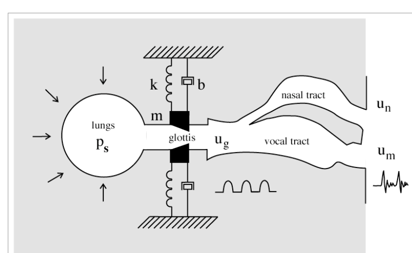


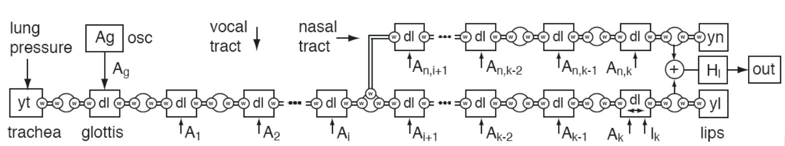

```
(patch ((zb (.Z :impedance *imped-zb*))
        (zt1 (.Z :impedance *imped-zt1*))
        (wt1 (.wtable *wt-data1*))
        (dl11 (.d-line :delay-length *length11*))
        (dl12 (.d-line :delay-length *length12*))
        ...)
   (-s zt (port dl11 0))
   (-> wt1 (-s (port dl11 1) (port dl12 0)))
   (-> (-s (port dl12 1) (port dl22 1) zb) da)
   ...)
```


DWG example (2): Voice production

- Kelly-Lochbaum type of transmission-line model of voice production model with controllable vocal tract shape
- Glottis waveform by Klatt model or inverse-filtered human source







DEMO!

Wave Digital Filter modeling (1)

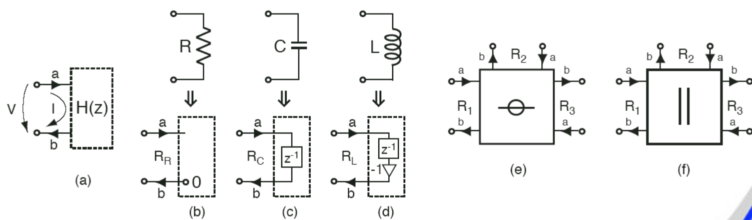
- Wave vs. Kirchhoff quantities are defined (in electical domain) by:

$$\begin{cases} a = V + RI \\ b = V - RI \end{cases} \leftrightarrow \begin{cases} V = (a+b)/2 \\ I = (a-b)/2R \end{cases}$$

- Instead of 'voltage' waves above, power normalized waves yield

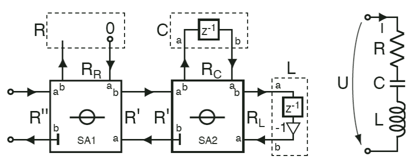
$$\begin{cases} a = (V + RI)/2\sqrt{R} \\ b = (V - RI)/2\sqrt{R} \end{cases} \leftrightarrow \begin{cases} V = (a+b)\sqrt{R} \\ I = (a-b)/\sqrt{R} \end{cases}$$

- WDF elements, (a) general one-port, (b) resistor, (c) capacitor, (d) indictor, (e) adaptor for series and (f) parallel connection:



Wave Digital Filter modeling (2)

- Example of a WDF circuit: series RCL connection



- The same model can be applied to a mechanical system by
  - inductance -> mass (force/acceleration)
  - capacitance -> spring (force/displacement)
  - resistance -> damping (force/velocity)
- ... or to an acoustical system by a corresponding mapping
- WDFs were originally developed for lumped system models
- WDFs include several special components
- WDFs have been generalized to multidimensional networks

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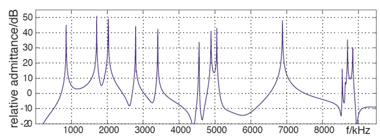
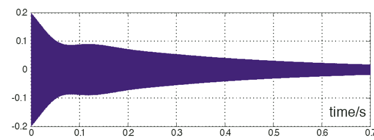
Example: Wave Digital Bell

- A bell recording is converted into a driving-point admittance, modeling lowest 10 partials, each conteining two modes very near in frequency
- 'Semiphysical' modeling since the real physical structure of the bell is not modeled



Spectrum of the bell sound



Envelope of the third partial

Example: Digital Waveguide Bell

- Based on inharmonic digital waveguide and extra resonators for creating beating partials (= mode groups)
- Very efficient computationally

DEMOS!

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Finite Difference Time Domain modeling (1)

- Discrete-time modeling of propagation and scattering using Kirchhoff variables (K-modeling)
- Second-order derivatives in the wave equation are approximated by symmetric second-order differences:

$$y_{tt} = c^2 y_{xx}$$

$$y_{xx} \approx -(2y_{x,t} - y_{x-\Delta x,t} - y_{x+\Delta x,t})/(\Delta x)^2$$

$$y_{tt} \approx -(2y_{x,t} - y_{x,t-\Delta t} - y_{x,t+\Delta t})/(\Delta t)^2$$

- => recursion formula (k = position index, n = time index):

$$y_{k,n+1} = y_{k-1,n} + y_{k+1,n} - y_{k,n-1}$$

- => after generalization to an arbitrary number of inter-connectd acoustic admittances Y_i

$$P_{\mathrm{J},n+1} = \frac{2}{Y_{\mathrm{tot}}} \sum_{i=0}^{N-1} Y_i P_{i,n} - P_{\mathrm{J},n-1}$$

See next page for a DSP formulation

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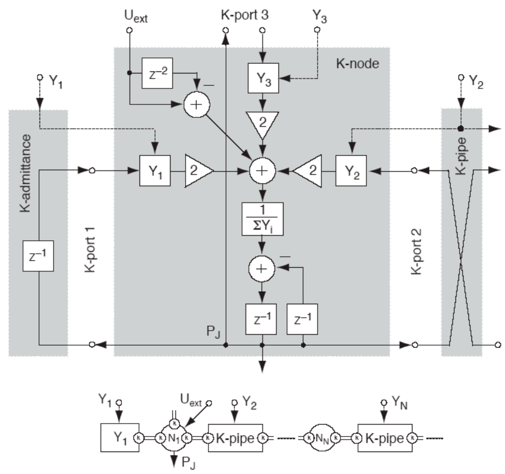
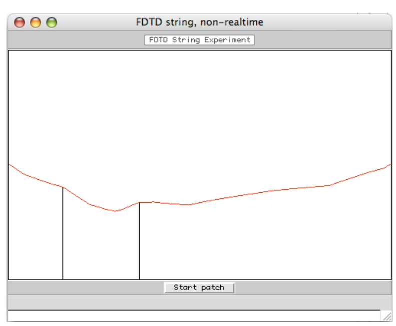
Finite Difference Time Domain modeling (2)

- DSP formulation of FDTD junction, *equivalent to the DWG junction before*, and its block-based abstraction



FDTD case: Visualization of string vibration

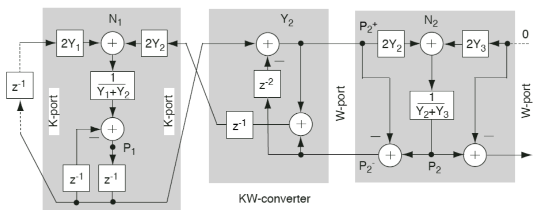
- An FDTD model of a vibrating string is computed and *visualized*
- Force inserted to the model is controlled by mouse
- Hard constraints are applied (such as string hitting a fret)



DEMO!

Interfacing DWGs and FDTDs

- K- and W-models are not directly compatible
- Converter element needed for mixed modeling

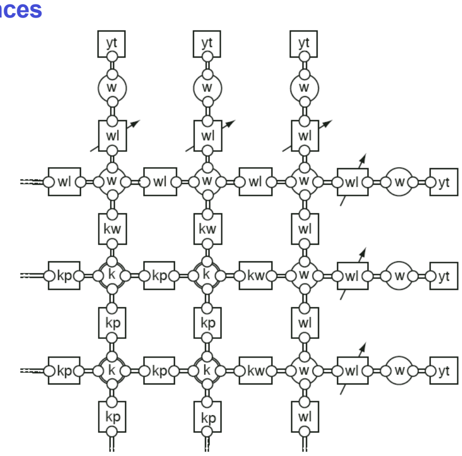


The KW-converter in the middle works as:

- a delay to the left (DWG -> FDTD) and
- as a delay-free pipe to the right (FDTD -> DWG)

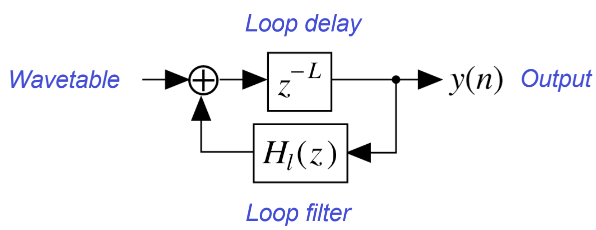
Modeling of 2- and 3-D mesh structures

- Example: part of a 2-D waveguide mesh with terminating impedances



Source-Filter modeling: Karplus-Strong string model

- **Simplest model, reduced from physics-based models to a single feedback loop with delay and loop filter for strinng damping**
- **By proper excitation and filter parameters generates high-quality string sounds**
- **Extremely efficient computationally**



DEMO!

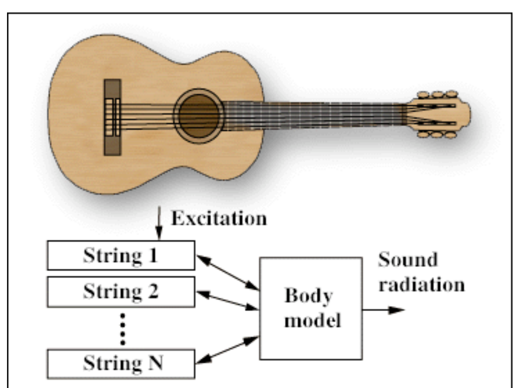
Source-Filter Modeling case: Extended Karplus-Strong model

- **Full model of the acoustic guitar, based on two-polarization string models, commuted body model, driven by high-level control software**

Examples by:
Mikael Laurson
Sibelius-Academy

Scale

Torroba



Summary

What has been discussed and demonstrated:

- Physics-based modeling paradigms
  - Digital waveguides
  - Wave digital filters
  - Finite difference models
  - Modal decomposition
  - Source-filter modeling
- Interrelations and mixed modeling
  - K- vs. W-modeling (Kirchhoff vs. wave quantities)
  - Mixed K- and W-modeling
- Real-time simulation and sound synthesis
- Modeling examples + DEMOS!

Thank you for your attention!

Special thanks are due to:

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Vesa Välimäki, Cumhur Erkut, Paulo Esquef, Jyri Pakarinen, Henri Penttinen, Hannu Pulakka, Lauri Savioja