Fast Multipole Boundary Element Method for Acoustic Impedance Boundary Value Problems

Seppo Järvenpää* and Pasi Ylä-Oijala

Electromagnetics Laboratory, Helsinki University of Technology
P.O. Box 3000, FIN-02015 HUT, Finland

Integral equation methods have been widely used to solve various time harmonic acoustic problems. For open-region problems the boundary element method (BEM) is an attractive method since the radiation conditions are automatically satisfied and the integral equations can be restricted to the surface of the object. BEM based on the traditional surface integral equation formulation, however, has three major numerical difficulties. Firstly, the solution is not unique at certain frequencies corresponding to the interior resonances of the object. Secondly, the integral kernels are singular as the field point coincides with the source point and advanced numerical techniques are usually required in these cases. Thirdly, a discretization of a surface integral equation results in a system matrix which is fully populated and expensive to store and solve. A direct solution of a dense linear system with N unknowns requires $\mathcal{O}(N^3)$ CPU time and $\mathcal{O}(N^2)$ computer memory. These numbers are prohibitively expensive for large scale problems. Iterative methods, which are very attractive with the finite element method, only reduce the CPU time to $\mathcal{O}(N^2)$. Since the system matrix of an integral equation method is dense, iterative methods are very slow for large scale problems and other methods, called fast methods, are usually required to speed up the calculations.

In this talk the aforementioned difficulties of the BEM are addressed in the context of exterior time-harmonic acoustic impedance boundary value problems. To avoid the internal resonance problem associated to the traditional Helmholtz integral equation formulation, a combined Helmholtz integral equation formulation, also known as a Burton-Miller integral equation formulation (BMIE), is considered. The numerical implementation is based on the Galerkin type discretization with the high order polynomial basis and testing functions. To avoid the numerical difficulties associated to the singular integral kernels, the singularity extraction technique is applied to compute the integrals in the singular and near-singular cases. In this method, the singular and near-singular terms are extracted from the kernel and integrated analytically. The remaining terms are smooth and can be integrated with standard numerical methods. In order to consider large scale 3D problems with a high number of unknowns, multilevel fast multipole algorithm (MLFMA) with an incomplete LU (ILU) preconditioner is applied to solve the matrix equation. With MLFMA both the required CPU time and memory can be reduced to $\mathcal{O}(N \log N)$, making the BEM available on a considerably larger scale problems. The numerical experiments demonstrate that the developed ILU preconditioned GMRES (generalized minimal residual) method for the BEM based on the BMIE formulation and discretized via Galerkin's method is well suited for an efficient application of the MLFMA.