Coefficient-Modulated First-Order Allpass Filter as Distortion Effect

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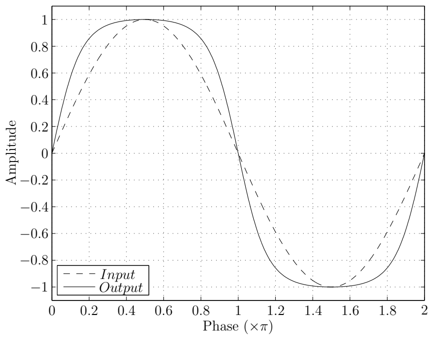
Outline

Source: `http://www.ibanez.com/images/artistFeature/_Ihsahn.jpg`

Introduction

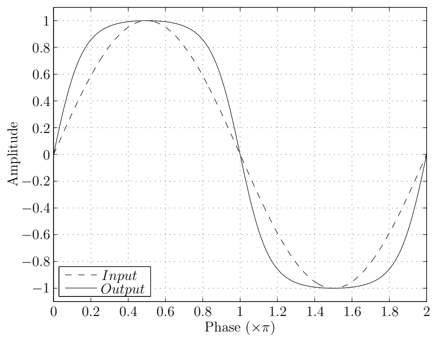
- ♭ Distortion is an essential effect especially in electric guitar playing
- ♭ Conventionally distortion circuitry modifies the signal amplitude
  - ♮ Hyperbolic trigonometric function
    - ♯ Nonlinearity often found in electric circuits
    - ♯ sinh (Yeh et al., 2007)
    - ♯ tanh (Huovilainen, 2004)
  - ♮ Chebychev polynomials (Gustafsson et al., 2004)



In figure: $f(x) = \frac{\tanh(2x)}{\tanh(2)}$

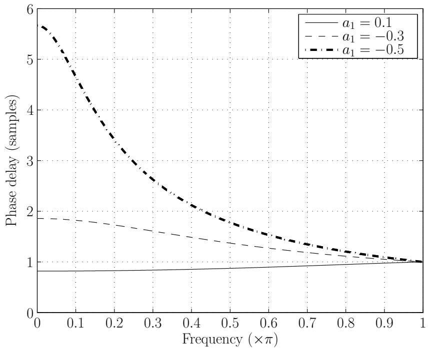
Amplitude Distortion by Phase Modulation

- ♭ Any signal can be represented at any time instant using amplitude and phase
  - ⇒ Nonlinear amplitude modification can be interpreted as a modification of the phase increment of the input signal!
- ♭ Phase modulation of an arbitrary signal by means of adaptive frequency modulation (AdFM) (Lazzarini et al., 2007)
- ♭ Direct AdFM approach not practical
  - ♮ Complex control logic
  - ♮ Requires a (long) delay line



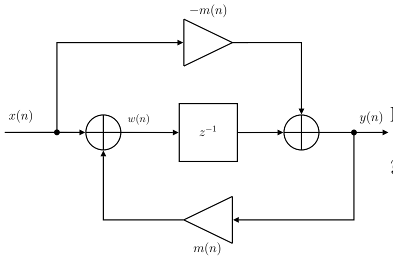
COEFFICIENT-MODULATED ALLPASS FILTERS

- An allpass filter modifies only the phase of the input signal
  - Frequency-dependent delay
- Allow the coefficients of a low-order allpass filter to be time-varying
  - ⇒ Time-varying frequency-dependent delay
  - Modification of the phase increment of an input signal
  - The resulting filter no longer allpass!



In Figure: Phase delay of a first-order allpass filter $H(z) = \frac{a_1 + z^{-1}}{1 + a_1 z^{-1}}$ for different values of a_1.

COEFFICIENT-MODULATED FIRST-ORDER ALLPASS FILTER



$$\begin{cases} w(n) = x(n) + m(n)y(n) & \text{and} \\ y(n) = -m(n)x(n) + w(n-1), \end{cases}$$

Expansion of $w(n)$ yields:

$$y(n) = -m(n)x(n) + (1 - m^2(n-1))x(n-1) + \sum_{k=2}^{\infty}\prod_{l=1}^{k-1} m(n-l)(1 - m^2(n-k))x(n-k).$$

- Time-varying first-order allpass filter used previously in modeling
  - nonlinear spring termination (Pierce and van Duyne, 1997)
    - Switching between two fixed values
  - tension modulation phenomenon (Pakarinen et al., 2005)
    - Phase delay at DC limited between zero and one
- Now $m(n)$ is not limited! Except with conditions for stability...

PROPERTIES OF THE PROPOSED FILTER

STABILITY ANALYSIS

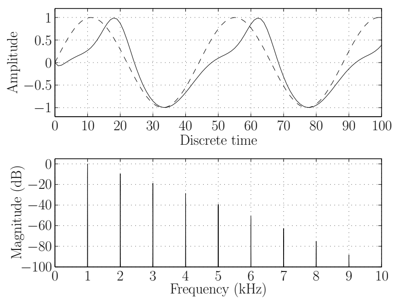
- Stability criteria of time-invariant recursive filters **NOT** applicable to time-varying filters (Laroche, 2007)
- Conditions for stability can be derived from the state-space representation of the filter
  - ⇒ For the proposed filter $|m(n)| \leq 1$

PHASE DELAY AT DC

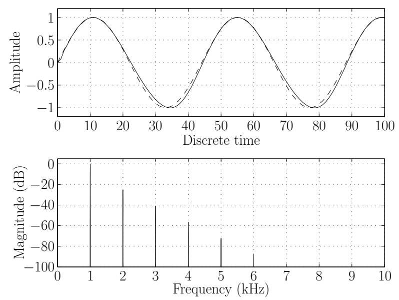
- $D(n) = \frac{1-m(n)}{1+m(n)}$ (Jaffe and Smith, 1983) ⇒ DC delay always nonnegative
- $m(n) = 1 \Rightarrow D(n) = 0$; $m(n) = -1 \Rightarrow D(n) = \infty$
- When phase delay at DC is large, the filter is highly dispersive
  - ⇒ unnatural artefacts not desirable in distortion effect when the input is a broadband signal

MODULATION SIGNAL CHOICE FOR ELECTRIC GUITAR PLAYING

- For a mild distortion light modulation and for heavier distortion more drastic
- The range of values $m(n)$ gets affects the resulting effect
  - ⇒ Example: input signal a 1000 Hz sine



$m(n) = 0.45 + 0.45x(n)$



$m(n) = -0.45 - 0.45x(n)$

HOW TO CHOOSE THE MODULATION SIGNAL?

- The input signal as is
  - Usually non-smooth ⇒ large distortion
- Lowpass filtered input signal
- Constant modulation signal, e.g., a sinusoid

DEMOS

Example 1

I Input signal

II Modulated by lowpass filtered input signal

III Modulated by a 800 Hz sine, $(-1, 0.6)$

Example 2

I Input signal

II Modulated by lowpass filtered input signal

III Modulated by a 800 Hz sine, $(-1, 0.6)$

CONCLUSIONS

- Amplitude distortion can be obtained by phase modulation
- Efficient implementation with a coefficient-modulated low-order allpass filter
- Coeffient-modulated first-order allpass filter tested
  - Pros
    - Computationally efficient
    - Freedom to choose the modulation signal
    - Possibility to be almost alias-free
  - Cons
    - Only one degree of freedom
    - Difficulty to choose the modulation signal?
    - Too simplified approach?

Demos available at: `http://www.acoustics.hut.fi/~jpekonen/Papers/dafx08/`

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