SONIFICATIONS AS MATHEMATICS TEACHING TOOLS

Robert Upson

Center for Science Education
Portland State University
PO Box 751
Portland, Oregon, 97201-0751, USA
jonnyq@scn.org

ABSTRACT

Six students between the ages of seven and thirteen participated in a lesson on visual graphing. The lesson for four students was supplemented with sonifications. Students found the sonifications engaging and fun. Recommendations for improvements to sonification enhanced curricula are given.

1. INTRODUCTION

Mathematics is the science of numbers, their order, interrelationships, operations, transformations, and generalizations. An examination of mathematical history reveals many forms of mathematics. The Greek mathematicians rebuked numerical mathematics due to the discovery of irrational numbers. This led to the development of a completely visual mathematics known as geometry [1]. In comparison, in many African cultures where there is little written record of knowledge, mathematics takes form in processes such as artwork, music, or proverbs [2]. Teachers sensitive to multiple learning styles [3] and multicultural education can benefit from inclusion of these various mathematical forms into their curriculum.

Teachers must also consider student outcomes when designing a curriculum. In order to maintain high standards of teaching and assist teachers in curriculum design, the Oregon State Department of Education, U.S.A., has developed a system of benchmarks for students to meet as they move through the education system. Mathematics benchmarks include the following Curriculum Goal headings: Patterns; Functions & Operations; Representations of Mathematical Relationships; Estimations; Processes & Strategies; Communication; and Organization of Data. These headings do not demonstrate a bias in learning styles, but in practice math has a strong visual component. Recommendations for the future of mathematical education do not prescribe visual math either [4].

Visual representations of numbers are created to make patterns or trends in a data set evident [5]**.** By embedding the data in the representation the clutter of the numerical symbols is hidden and interval relationships possibly become apparent. To the veteran mathematician or scientist nothing seems more routine. We forget that learning how to interpret, create, or draw conclusions from graphical representations are not nominal tasks when first introduced. As discussed in this study, some students seem naturally prone to this form of analysis while others have a very difficult time learning how to create and read visual graphs.

Sonifications, as information carriers in non-speech sound mediums [6], require numerical transformations for construction and present an exercise in mathematics. Music and sound are of interests to many elementary and secondary students, therefore, aural mathematics seems to offer an interesting opportunity for mathematics teaching. The primary aim of this work is to explore how sonifications may be used to help students learn math. While considerable work has been completed on the applicability of sonifications for data analysis, I have found none examining the use of sonifications as a general math teaching tool. Secondary aims of this work include providing ground-based information on human computer interaction with sonifications, curriculum design issues invoked when using sonifications, and evidence of improved metacognition skills when students learn a combination of representation schemes.

2. THEORETICAL FOUNDATIONS

2.1. Metacognitve Approaches to Teaching and Learning

Many researchers agree that allowing a student to personalize his or her learning experience, to some degree taking control of the learning framework, is one path to educational success [7][8][9][10]. These studies show that presenting mathematical concepts in several representations at once can improve understanding. Interrelationships may develop among representations presented concurrently. The student may also be taking ownership of his learning by having a choice of representation [8].

Davis [6] reports of an amateur musician who naturally converted musical notes to numerical fingerings of his instrument, the clarinet. Sonic-numeric transformations of complex data sets, made possible by computers, may offer the musically oriented student an opportunity to succeed in the predominantly visual field of mathematics.

2.2. Society and Technology

When a calculator can perform rudimentary calculations what place does the traditional math curriculum, primarily operational algorithms, have in the school? Technology's changing role in society is being realized in changing curricula [11][12]. The new skills needed to be valuable in the workplace raises concern among parents, school administrators, and educational researchers about the traditional teaching aims [4] [13].

The Oregon benchmarks answer this concern by offering wide interpretation of what constitutes mathematics. Rather than being task based, it is process based [8]. It demonstrates understanding that the problems faced by today's students will not be solved with simplistic conclusions, or with prescribed treatments. Sonification tools can offer alternative customized environments in which students may explore and analyze problems and relationships.

3. INITIAL EXPERIENCES WITH SONIFICATIONS IN THE EDUCATIONAL SETTING

3.1. Secondary Science Teacher Training Program

During a four-week intensive secondary science teacher training program I introduced sonifications as one possible data analysis technique. We sonified parameters such as temperature, wind speed, humidity, solar UV, and rain over time (10 minute sampling rates for data acquisition). The sonification scheme was similar to that described below with the exception that multiple dimensions were presented concurrently on some occasions. Of twenty-five participants about five showed interest in the idea of the technique. However, only one inquired further about applications or introduction in her classroom. This teacher was interested in sonifications for a visually impaired chemistry student.

Colleagues of mine and the lead educator of the training program found the technique fascinating but seemed somewhat reliant on visual representations to draw conclusions about trends in the data that was sonified.

3.2. Secondary School Math Teachers

When demonstrating the technique to practicing secondary math teachers at their school I was entertained, but generally dismissed. One teacher commented that the sound would be disruptive to class. The math teachers at this school have a strong community. They share curricula and integrate class content with other subject areas. Standardized testing scores of their students are well above average. It seemed that sonifications for data analysis were not viewed as a technique capable of helping these teachers maintain high test scores for their students.

3.3. Middle School Science Class

I presented the use of sonifications to analyze patterns in weather data to a class of thirty 7th and 8th grade science students. The visual computer interface was projected onto a screen as I manipulated the sonification program and played the sonification from a laptop computer. Sonifications were created from weather data, as in section 3.1. I highlighted patterns they might encounter (e.g. rising and falling UV over the course of a day) and altered the mappings based on student interest (e.g. trumpet vs. piano voice). Some students had difficulty hearing the sonifications due to proximity. Others had problems due to talking by classmates. Several students approached me after the short lesson to listen to more sonifications and inquired how they could use the program again. One student said, “So it’s a sound effects machine.” Another said, “It sounds like Mario Brothers,” a comment heard several times since.

4. CURRICULUM DESIGN

After the initial experiences described above, and others, I was careful in my choice of goals during design of a curriculum. I thought it best to create a lesson that matched current teaching benchmarks. This precluded any lesson that would use sonifications for data analysis since aural representation and interpretation skills are not measured or available during Oregon State testing. Rather, I decided to focus on using sonifications as supplements to graphing.

Before designing lessons for a large classroom I chose to begin study in personalized, one-on-one teaching sessions. This allowed in-depth questioning before and after a session to assess what students learned, and most importantly, how they incorporated sonifications into their cognitive frameworks. I wanted to keep the students active, either talking, using a synthesizer keyboard, or drawing graphs during the lesson. In this way the student might benefit from maintaining a social context, or dialog, with either real objects or beings during the session [10][14][15]. The session was not meant to be a computer tutorial on how to graph or use sonifications. Besides being too prescriptive, I was aware of the faults of programmed teaching developed for television-based education [16]. It is important to remember that sonifications are at best tools for teaching and cannot substitute for the teacher. To assist students in making the connection from numerical to either aural or graphical representation I designed activities that did their best to explicitly demonstrate processes behind such transformations [17].

The completed lesson plan began by informing students of the goals. I used Oregon population and Portland, Oregon weather data (temperature and precipitation) as example data sets since both seemed intuitively comprehensible to even the youngest student. Students were first introduced to simple numerical data sets and their conventions. Next, more complex real-world data sets were presented in numerical form. Sonifications of elementary and complex data sets followed as a first example of data transformation. A synthesizer keyboard was available for students to attempt to mimic the sonifications and experiment with creating sonifications from data sets themselves. Musical scores were presented to show alternative visual representations for a sonified data set. Examples of other visual data representations used in everyday life were provided in the form of newspaper and magazine charts and graphs. This led the student into activities on creating and interpreting graphs using Excel graphing functions on the computer and a drawing pad.

5. EXPERIMENTAL DESIGN

Six participants, age seven to thirteen, were identified by local elementary and middle school math teachers. Two participants completed the control lesson and four completed the experimental lesson. Each lesson lasted approximately thirty minutes with about fifteen minutes of questions and activities prior to and immediately after the lesson for assessment purposes. Sessions were audio taped for later analysis. Several assessment questions allowed use of the synthesizer and/or drawing pad.

The study was initially designed for a control and experimental group differentiated by inclusion or exclusion of sonification supplements to the lesson. Students were to be selected based on a limited introduction to graphical mathematics and assessed for improvement compared to students in the control group. In practice, no students could be found with the minimal level of graphing ability, even at the first grade level (age 7). It appeared that understanding of numerical increases is learned in unison with visual concepts of high and low. Visual representation and interpretation is simply too embedded in the culture to be completely screen out. Indeed, that visual representations work very well in many scenarios, for many people, explains their widespread use. In addition, several large and small departures from the lesson, as described in section7, preclude drawing scientific conclusions from the experiment.

6. SONIFICATION SCHEME

Sonifications were created using a custom JAVA v.1.1.8 application. Data dimensions were read from text files and outputted in the .mid format type 0. The .mid format allows 128 discrete points for each frequency and volume attribute. The 20 tail points were not used, which allowed 88 discrete points in the sonifications. The JAVA application scans for high and low data values in individual data dimensions and then scales each data dimension within its own domain. Each data point is then rounded to a whole number and mapped to frequency or volume (all other attributes uniform). Up to six data dimensions were presented simultaneously in preliminary studies. During the student lessons only one data dimension was sonified. Sonifications were played over built-in stereo laptop computer speakers and processed by an XpressAUDIO 16-bit soundcard.

7. PARTICIPANT PROFILES

7.1. Control Participants

David is an 11 year old attending an urban middle school. He uses computers for accessing the Internet and has one at home, but it doesn't work. He likes math and thinks of it as "numbers" and that sometimes it can get confusing. He also thinks that "math" and "computers" are the same thing. He does not play any instruments but listens to all kinds of music including rap and rock. Prior to the lesson he was capable of interpreting quantities and patterns in tabular and graphic data.

After the lesson, he was the only student in the study to construct a conventional graph from tabular data, as defined by uniform axis scales with the time variable on the horizontal axis and the data parameter on the vertical axis. He also seemed most capable of understanding how to collect data. He remarked that he thought the lesson was interesting. He liked drawing the graphs and thought that computers are useful in creating graphs. He was interested in a follow-up lesson. When asked what he learned during the lesson he wrote "about graphs" and the "population of Oregon".

Amanda is a 12-year-old 7th grader attending a suburban middle school. She also has a broken computer at home and uses the ones at school to "get ideas", work on school projects using word processing programs, and to access the Internet. She thinks math is "hard at first" and that you have "to like it and do your best." She enjoys open-ended math problems: those which use "hands-on things" because "I get to think of different ways to solve a problem, I can think for myself on it, use objects to do it." She says she uses math when baking with her mother or comparing prices at the store. She is aware of both her parents using math at their work, mostly for measuring. She thinks mathematicians think different than others, "going beyond what some people might, they might do a lot of extra stuff to figure it out." She played clarinet the previous year and was able to figure out the "buttons" from the score by the end of the year. She listens to pop and hip-hop music anywhere except at school on her portable.

Prior to the lesson Amanda did not see a pattern in the tabular data, although she recognized it was increasing. She did not demonstrate thorough understanding of axis scales on the pre-lesson graph. After the lesson she created a conventional graph with the exception that her vertical axis increased in value moving down the page. As she was describing her thoughts during the creation of the graph, and during her description of a trend for a second set of data, she demonstrated a strong ability be critical about her steps. Several "oops, wait, did I do that right? Did I put that in the right spot, yeah it's in July." She was "not sure" about other uses of computers for math after her lesson. She found the graphing interesting and was interested in a second session. She learned that "mathematicians make things look complicated, but when you put them in a different form it isn't that complicated," and "you can use graphs for charting information".

7.2. Experimental Participants

Ward is an 11-year old in the 6th grade attending an urban middle school. He has a computer at home that he uses to play games, access the Internet, and sometimes for homework. He likes math, especially geometry, and sees it as "adding, subtract, multiply, and all the basic, you know, math". He thinks his brother uses math at work, because he works with computers, and thinks he will use math "when 16, or 17, and the computer messes up, if working at a fast food place, and I have to do it myself in my head". He thinks mathematicians think differently than others, but he is not sure how. He doesn't listen to music and doesn't play an instrument. He was able to interpret and make crude predictions from the pre-lesson tabular and graphical data.

Ward described the graph he created after the lesson as a "behavior over time graph". The vertical axis was not scaled, rather, simply a progression of values in their tabular order. His horizontal axis was progressive in months thus creating a linear curve from data that otherwise would have graphed up, then down. He spoke of his graph out loud, saying he didn't think it was right and that his teacher had not taught him how to graph data, "going up, then going down." He said he liked the graphs and liked the sounds and he was interested in another session. Ward wrote that he learned "sine graphs increase then decrease," and that "if a graph is pulled or stretched, it is the same graph."

Damen is a 7-year old attending an urban elementary school. He uses computers to play games, paint, and spell words at home. He doesn't know anyone who uses math, but knows that math is "5 times 6 is 30", and "take away, plus, and um, times". He thinks he will use math when he grows up. He admits some of his classmates like math and others don't. He thinks they all like to play music and computers. He plays drums and piano, and listens to children's songs like "Twinkle Twinkle Little Star".

Damen cannot yet read written English or large numbers well. I had to alter the tabular numbers to a smaller scale for him. He was able to recognize relationships in graphs prior to the lesson, but did not seem capable of reading the quantifications they presented. Oddly he recognized one of the graphs from a newspaper as a "doctor graph" since the graph was in the same form as one he saw at the hospital. During the lesson Damen became tired and asked to stop. He wished to continue later so we met in four days.

In continuing the lesson I created a "score" of Twinkle Twinkle Little Star in tabular form. He enjoyed playing this game (playing the song from the table on the synthesizer) and was even able to create a traditional graph from the score. Later saying "I only like this, the piano graph. I mean when you play a song and it shows on the graph, and you do a graph…" although he also said, "I don't like graph, I only like math." He was also recognizing patterns in the graph of the notes, "cause look, see it copies, it's that and that". He was interested in another lesson.

Fred is a 12-year old attending an urban middle school. He likes computers for games and graphics but is not allowed to use them at home. He thinks math is fun and commented, "without math there is nothing". He uses math for comparing things and being efficient. He is certain he will use math in any of his several aspirations when he grows up. He thinks mathematicians and scientists ask questions and that they can think differently than others. To Fred, music is "another form of communication besides math and speaking". He listens to music a bit (definitely doesn't like "London Bridge"), makes noises with his mouth, and plays the harmonica, flute, and bagpipes. Fred incorporated a lot of history into his interpretation of the tabular population data I asked him to explain prior to the lesson. He also interpreted the pre-lesson graphical temperature and precipitation data in terms of seasonal weather patterns, aptly describing quantitative measurements presented.

During the lesson Fred was enthusiastic about playing the keyboard, often anticipating when I would ask him to try to play data sets. At one point he was willing to play two data dimensions simultaneously, temperature and precipitation. The volume and frequency mappings used for computer sonifications make sense to him. In other words, he could hear the mappings, as demonstrated in his descriptions of them and his skill at playing what he heard. He was so sure of his capability to create graphs that I skipped parts of the visual representation lesson.

After the lesson, however, he drew a graph with time on the vertical axis and the dependant variable on the horizontal axis with no consistent scale (simply progression of data values). He spoke of being aware of mistakes in his graph, particularly in plotting the individual points. He liked the lesson and would not mind another. He wrote that he learned, "many different views toward math in the sense of graphing."

Shane is a 13-year old 7^{th} grader at a suburban middle school. He thinks math can be "easy or challenging, depending on what I'm doing." He thinks that everybody uses math in some way, and is therefore certain he will use it when he grows up. He thinks mathematicians "might make things more complicated than they seem, but not really." He feels pretty comfortable with computers and uses them for homework, games, and the Internet at home and at school. He thinks computers are "part of the future". He has been playing music for three years--trombone and bass guitar in a jazz band. He listens to pop, jazz, rap, and classical music, "though some of it is boring," and also listens to "bands, not just singing, the musicians and the drumming". He was able to interpret both the tabular and graphical representation prior to the lesson, and make good conclusions about their trends.

He is the only participant who did not hear a pattern in sonified seasonal temperature data. He is also the only student who said he would not be interested in a second lesson. The graph he drew after the lesson is extremely similar in design to the one created by Fred. Also like Fred, he made a good estimation of future trends in data presented in tabular form. When asked how computers can be used for math he said for graphing and gave an example of a graph that could be created. He said he found the sounds "pretty interesting, but the graphs kinda boring", and wasn't sure if he learned anything, except "I learned stuff about sound."

8. DISCUSSION

As demonstrated by post-session graphing exercises, the experimental group performed poorer at conventional graph drawing and analysis than the control group. This may be due to the fact that the control group received more training in visual graph creation [18].

Shane, Ward, and Fred would have been more engaged with real-world examples of sonifications in use for analysis and problem solving. These students already had a fair amount of experience with graphing, and the sonifications were most interesting for concepts of scaling and pattern recognition, not as alternative numerical transformations aids. Since the sonifications did not immediately appear as powerful at communicating information as visual graphics I was met with resistance in teaching them. These students, some of whom were in preparation for complex systems dynamics curriculum, may be more receptive to multi-dimensional sonification examples than the rudimentary sonifications in which I played for them. One "ah-ha" moment occurred when I played a quick sonification and a slow sonification of the same dataset for Ward. I then showed him a visual graph and altered the shape of its border. He understood the concept immediately--that the data does not change, but the representation does.

Damen was deeply engaged when working with the synthesizer keyboard and computer graphing application to play a children's' song. A simple sonification training scheme would have been valuable to help Damen make the transition from synthesizer to computer sonifications. The synthesizer worked well for data transformations involving small numbers (less than 18 for the number of keys on the synthesizer), but it was hard for him to understand what to do with larger numbers or how they were transformed to sounds. Damen began the lesson with little concept of data organization so it was a small success to find him reading and playing a song from a graph at the end of our lesson.

Even extensive preparation for the lesson presented to the students did not provide me with enough activities utilizing sonifications. It is necessary to design more activities to fit a wide range of student developmental needs. As stated above, an introductory sonification activity making transformations more explicit is important and most likely beneficial to even the most casual user of sonifications [19]. More game-like activities would be helpful in teaching younger students like Damen. Activities that relate sonifications to music, or music theory, would be of interest to older students like Shane. Examples of sonifications being used to solve real-world problems would also be helpful.

Improvements in sonification software are also necessary. Working with data sets connected to the students' interests is ideal. However, converting these data sets, often found in multiple formats online, to sonifications was not possible in single sessions of limited time. A software application that could quickly map multiple data set formats to a sonification would be very helpful.

An interface with the ability to display both an audio and visual graph simultaneously, such as TRIANGLE [20], could improve the transparency of data transformations for the student [10]. The learning potential sonification offers would be improved by adding more virtual buttons, levers, and knobs to make it possible for the student to explore the data set more easily [21]. This would also allow the student to modify the aural environment to his or her personal mapping preferences [22]. The ability to manipulate sonic attributes in real time is important and could exclude the use of the .mid format in future applications. The lack of psychophysical scaling in the data mapping [23] did not seem an issue for most students since detailed analysis of the sonifications was not a concern, only general patterns and transformation possibilities. Shane is the only student who had difficulty interpreting simple patterns in a sonified data set.

9. CONCLUSIONS

The use of sonifications in the educational setting offers a provocative alternative vision to traditional mathematics education goals and strategies. More work needs to be conducted on developing sonification activities for various student development levels. Sonification tools that allow instant data mapping is required for wider use in the educational setting. Sonification offers mathematics education a pathway toward meaningful instruction in a technologically influenced, society in transition.

10. REFERENCES

[1] David M. Burton, *The History of Mathematics: An Introduction*, 4th ed., Boston, Mass.: WCB/McGraw-Hill, 1999.

[2] Claudia Zaslavsky, “Africa Counts,” Boston, Mass.: Prindle, Weber, and Schmidt, 1973.

[3] Linda Campbell, Bruce Campbell, & Dee Dickinson, *Teaching & Learning Through Multiple Intelligences*, Needham Heights, Mass.: Simon & Schuster, 1996.

[4] National Research Council, *Everybody Counts: A Report to the Nation on the Future of Mathematics Education,* Washington D.C.: National Academy Press, 1989.

[5] F. Bastide, “The iconography of scientific texts: principles of analysis”, in *Representations in Scientific Practice*, Michael Lynch and Steve Woolgar, eds., Cambridge, Mass.: MIT Press, 1990, pp. 187-231.

[6] International Community for Auditory Display (ICAD), *Sonification Report: Status of the Field and Research Agenda*, ICAD, 1999.

[7] R. Davis, *Learning Mathematics: The Cognitive Science Approach to Mathematics*, New Jersey: Ablex, 1984.

[8] R.D. Pea, “Cognitive Technologies for Mathematics Education”, in *Cognitive Science and Mathematics Education.* A.H. Schoenfeld ed., Hillsdale, N.J.: Lawrence Erlbaum Assoc., 1987, pp. 89-122.

[9] A.H. Schoenfeld, “What’s All the Fuss About MetaCognition”, in *Cognitive Science and Mathematics Education.* A.H. Schoenfeld ed., Hillsdale, N.J.: Lawrence Erlbaum Assoc., 1987.

[10] ***Computing and Mathematics: The impact on secondary school curricula: report of a conference***, James T. Fey, ed., College Park, Maryland:National Council of Teachers of Mathematics, 1984.

[11] Joe Crosswhite, “A Mathematics Educator’s Perspective”, in *Cognitive Science and Mathematics Education.* A.H. Schoenfeld ed., Hillsdale, N.J.: Lawrence Erlbaum Assoc., 1987.

[12] G. J. Papagiannis, C. Douglas, N. Williamson, and R. LeMon, *Information Technology and Education: Implications for Theory, Research, and Practice*, International Development Research Center, Florida State University, MR161e, July 1987.

[13] Shoshana Zuboff, *In the Age of the Smart Machine: The Future of Work and Power*, New York:Basic Books, 1988.

[14] G.M Bowen, W-M. Roth, M.K. McGinn, “Interpretation of Graphs by University Biology Students and Practicing Scientists: Toward a Social Practice View of Scientific Representation Practices,” in *J. of Rsch in Sci. Teaching*, vol. 36, no. 9, pp. 1020-1043, 1999.

[15] S. Papert, “Computer Criticism vs. Technocentric Thinking,” in *Educational Researcher,* vol. 16, no. 1, pp. 22-30, Jan/Feb 1987.

[16] Richard E. Clark, “Reconsidering Research on Learning from Media,” *Review of Educational Rsch.*, vol. 53, no. 4, pp. 445-449, 1983.

[17] J.G Greeno, “Instructional Representations Based on Research about Understanding”, in *Cognitive Science and Mathematics Education*, A.H. Schoenfeld ed., Hillsdale, N.J.: Lawrence Erlbaum Assoc., 1987, pp. 61-68.

[18] W-M. Roth and M.K. McGinn, “Graphing: Cognitive Ability or Practice?” in *Science Education*, vol. 81, pp.91-106, 1997.

[19] J.H. Flowers and T.A. Hauer, “Musical Versus Visual Graphs: Cross-Modal Equivalence in Perception of Time Series Data”, *Human Factors*, vol. 37, pp553-569.

[20] John A. Gardner, Randy Lindquist, and Steve Sahyun, “TRIANGLE: A Practical Application of Non-Speech Audio for Imparting Information,” in *Proc. of ICAD ‘96*, Palo Alto, California, Nov. 2001.

[21] N.J. Castellan Jr., “Evaluating Information Technology in Teaching and Learning,” in *Behavior Research Methods, Instruments, & Computers*, vol. 25, no. 2, pp. 233-237, 1993.

[22] Bruce Walker, Gregory Kramer, and David M. Lane, “Psychophysical Scaling of Sonification Mappings,” in *Proc. of ICAD*, Atlanta, Georgia, April 2000, pp. 99-104.

[23] J.G Neuhoff, G. Kramer, and J. Wayand, “Sonification and the Interaction of Perceptial Dimensions: Can the Data Get Lost in the Map?” in *Proc. of ICAD*, Atlanta, Georgia, April 2000, pp. 93-98.