Modeling of long and complex responses using Kautz filters and time-domain partitions

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A Transcription of the Title

Complex responses – long duration, high sample rate, complex time-frequency characteristics: distributed onset, high mode/resonance density, variable decay rates – in our case, measured room impulse responses

Kautz filters – a special class of pole-zero filters, originating from rational orthonormal basis representations – favorable properties due to orthogonality and the transversal (allpass) structure

Time-domain partitions – splitting the modeling task into manageable portions, a time-domain approach using 1) interlaced (polyphase) decomposition or 2) sequential segmentation of the target response

Motivation and premises

- The need for transfer function models for complicated systems such as room impulse responses
- Traditional choices: finite or infinite impulse response (FIR/IIR) linear time-invariant (LTI) filters using conventional structures and design methods
- Harsh reality:
  - FIR: high filter orders are required due to the nature of the target responses – a practical burden
  - IIR: in addition or consequently, potential stability issues – or simply, impossible to construct – also a principled problem
- Kautz filters: provide efficient high-order IIR filters for many purposes with considerably less computational complexity compared to FIR filters
- For really tough cases, such as concert hall impulse responses at high sample rates/full band-width: various subband techniques, partial artificial reproduction and/or the proposed time-domain partitions

Kautz filters in a nutshell

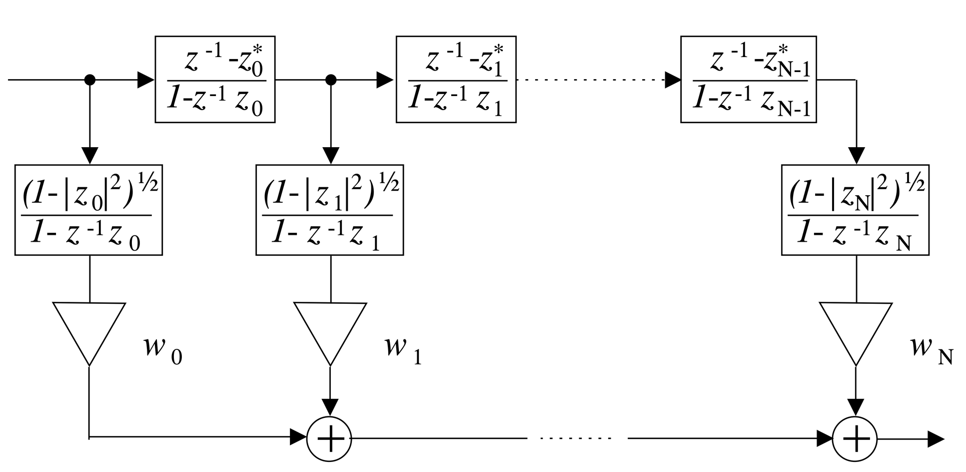
The generic form of a Kautz filter is given by the transfer function

$$\hat{H}(z) = \sum_{i=0}^{N} w_i G_i(z) = \sum_{i=0}^{N} w_i \left(\frac{\sqrt{1 - z_i z_i^*}}{1 - z_i z^{-1}} \prod_{j=0}^{i-1} \frac{z^{-1} - z_j^*}{1 - z_j z^{-1}} \right)$$

and it is specified by two sets of parameters

- Poles $\{z_j\}_{j=0}^{N}$, $|z_j| < 1$, define the orthonormal (Kautz) functions $G_i(z), i = 0, \ldots, N$ – and the filter order
- Tap-output weights $\{w_i\}_{i=0}^{N}$ – in our case, orthogonal (Kautz-Fourier) expansion/projection coefficients

The recurrent form of the basis functions $G_i(z)$ result in the depicted transversal structure:



The Kautz filter – for complex conjugate pole pairs, a modified real Kautz filter structure is used. More familiar special cases of the Kautz filter:

- For $z_i \equiv 0$ it degenerates to an FIR filter
- For $z_i \equiv a$, $-1 < a < 1$, it is a Laguerre filter

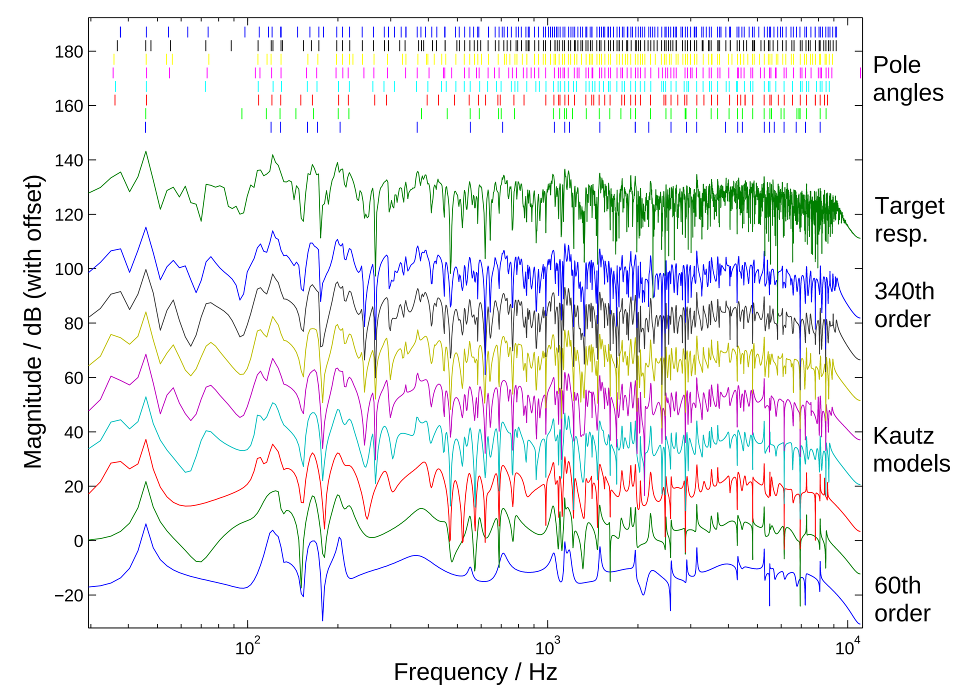
Sales talk – why Kautz filters?

- Efficient pole-zero modeling using FIR filter-like structure and design
- Kautz filter techniques using our pole generation method – challenges conventional IIR filter design
- Unconditionally stable, good numerical properties due to the transversal (allpass) structure
- Trivial model reduction/modification schemes due to orthogonality

Basic design steps

- Generate candidate pole sets (using our BU-method) w.r.t. the target response and desired model orders
- Possible variants (of the BU-method) using combined warping and frequency-zooming techniques
- Evaluate the Kautz-Fourier weights, inspect the result
- If needed, modify the pole set: prune, tune, attach, cluster – re-evaluate the weights, construct the model

Example I: Kautz models for a measured room impulse response (sample rate 22050 Hz, 8192 samples, 371 ms):



- A straightforward demonstration of the BU-method
- Kautz models, orders 60–340, in steps of 40
- Intermediate Bark-scale warping ($\lambda \approx 0.65$) is used to emphasize low-frequencies (warped BU-method)

Utilizing two time-domain partitions

- Impossible dimensions: for example, 100 000 samples and more than 1000 relevant resonant frequencies
- Solutions: subband techniques, artificial reproduction
- Alternative or additional methods: following partitions

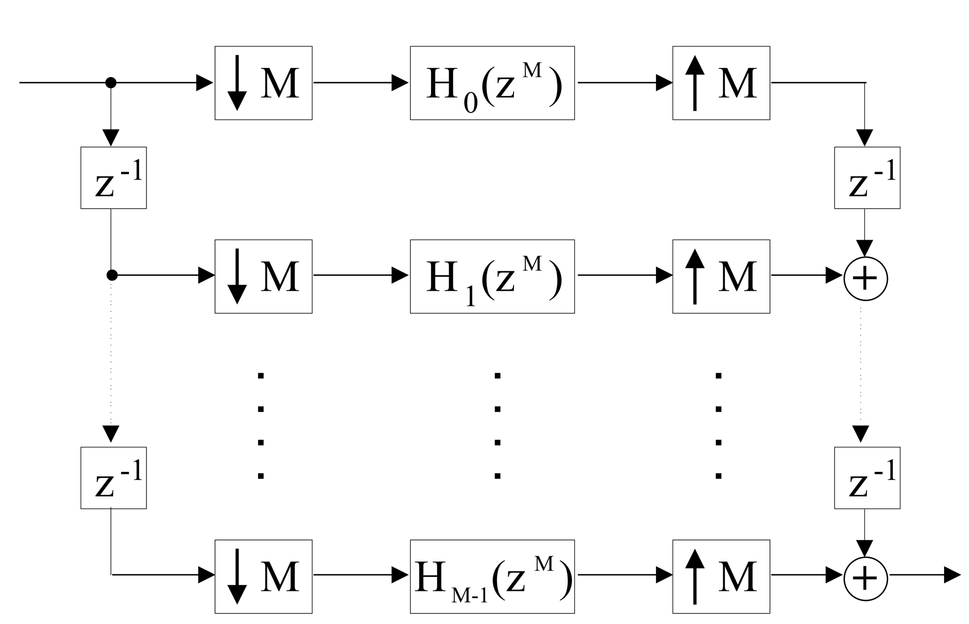
Polyphase Decomposition

The Mth order polyphase decomposition of an impulse response $h(n)$, $n = 0, \ldots, NM$, is given by

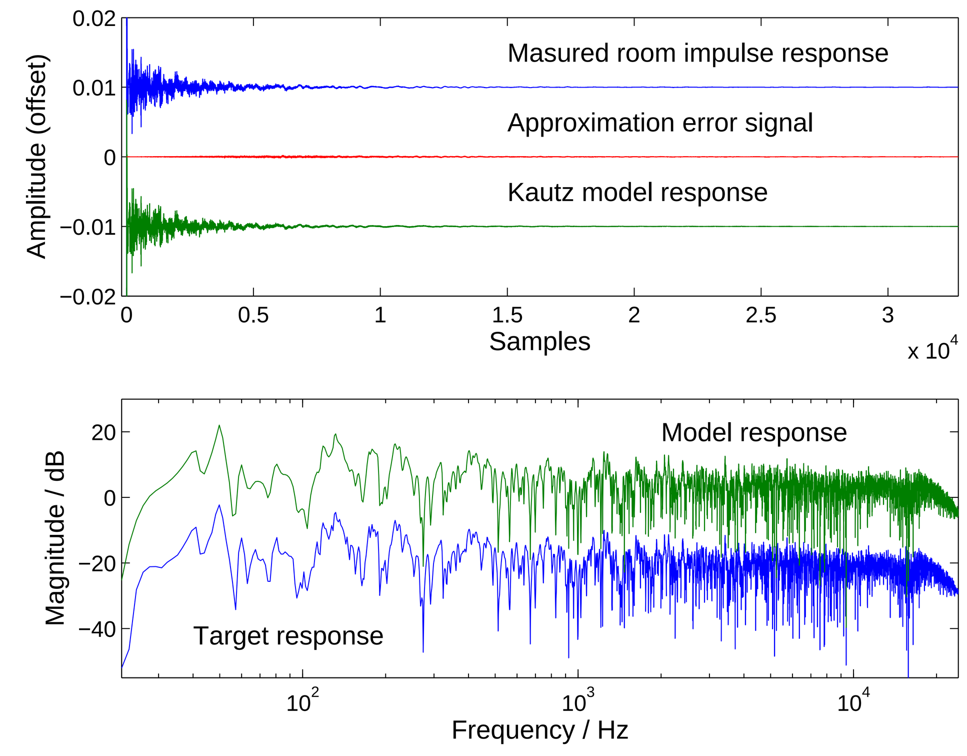
$$H(z) = \sum_{k=0}^{M-1} z^{-k} H_k(z^M), \quad H_k(z) = \sum_{n=0}^{N-1} h(Mn + k) z^{-n}$$

The component FIR filters $H_k(z)$ are then simply approximated by Kautz filters:

- Design the Kautz sub-filters w.r.t $h_k(n)$
- The construction rely on "waveform matching"
- The required sub-filter order N_k is typically $\approx N/10$



Example II: a measured room impulse response (44.1 kHz, 32768 s.), approximated by an 64×60th order Kautz model:



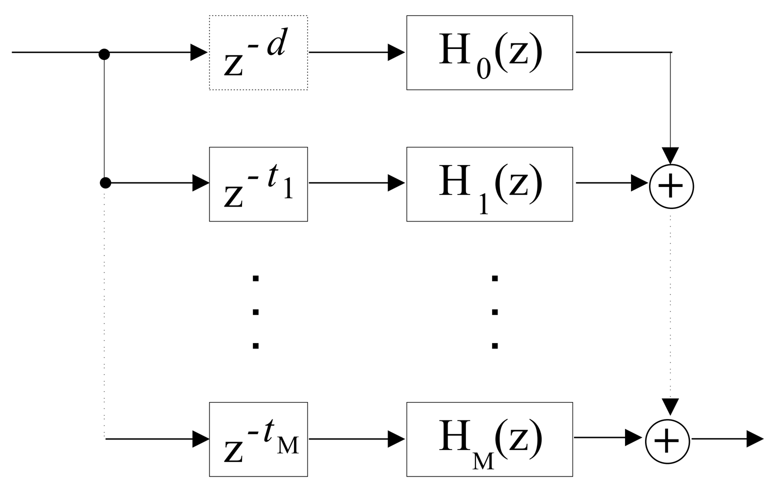
- The implied sub-signal length is 512 samples
- A fixed sub-filter order 60 – could be optimized
- The normalized square error is 0.0024

Sequential Segmentation

A given target response may also be partitioned into successive segments corresponding to the transfer function

$$H(z) = \sum_{k=0}^{M} z^{-t_k} H_k(z), \quad 0 \le t_0 < t_1 < \ldots < t_M < N$$

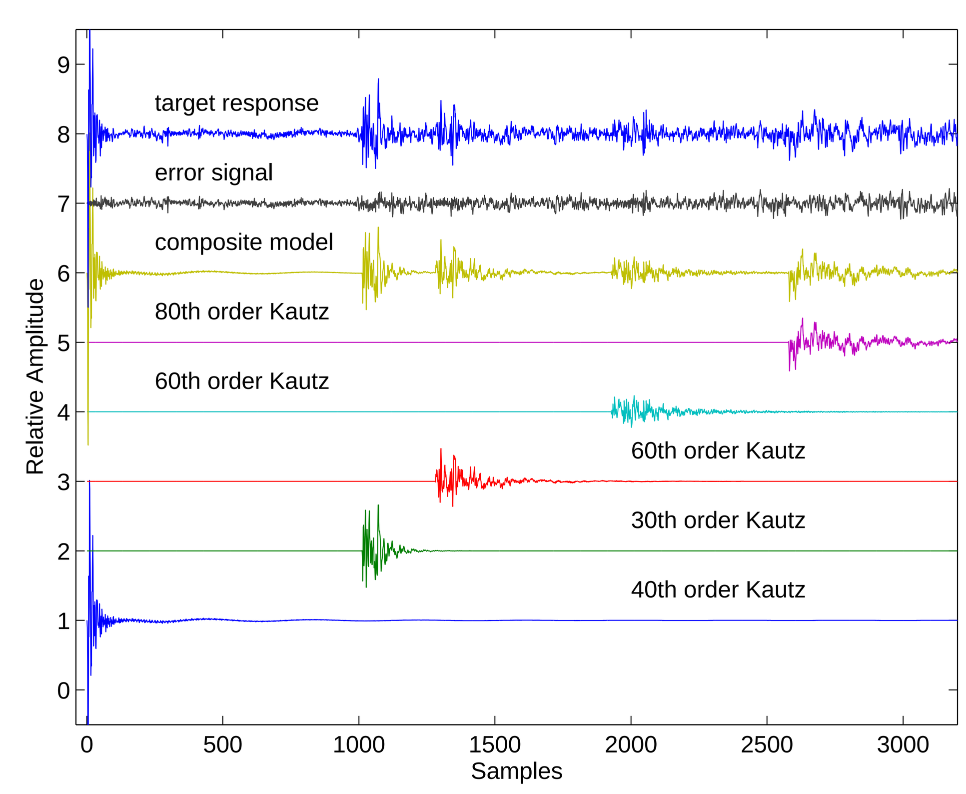
or schematically as below ($d = t_0$ is the initial delay):



The component FIR filters $H_k(z)$ are then once more approximated by Kautz filters, taking into account the possible IIR leakage:

- Choose the partition, isolate the target response
- Construct the Kautz model of a desired/sufficient order
- Subtract the possible "overflow" of the Kautz model responses when forming following target responses

Example III: the early part of a concert hall impulse response modeled by a chosen partition of Kautz models:



- Partition into relevant events – early reflections?
- A low-order model – exact match by increasing order
- "Parametric" control of the early response?

Conclusions and Remarks

- Kautz filter techniques are proposed as a genuine alternative to traditional FIR/IIR filter design
- Two simple time-domain partitions are used for unwrapping long and complex responses
- Just an idea – many open aspects, and possibilities?