# Exercise 10 (Chapter 9)

1. (a) Calculate the *absolute condition number* (sensitivity to errors) of matrix addition, $\mathbf{A} + \mathbf{B} = \mathbf{C}$. (Tip: It is sufficient to consider an error in an element of $\mathbf{A}$ and assume $\mathbf{B}$ is accurate.)

   (1 points)

   (b) Calculate bounds for *relative error* for the inner-product of vectors, $\mathbf{a}^H\mathbf{y} = c$ and matrix multiplication $\mathbf{A}^H\mathbf{B} = \mathbf{C}$, when using floating point operations.

   (2 points)

2. (Demonstration) Show that the $\ell_2$-norm of a matrix $\mathbf{A}$ is its largest eigenvalue. What is the $\ell_2$-norm of $\mathbf{A}^{-1}$?

   (0 points)

3. (Research question) The filter of linear minimum mean square error (LMMSE) estimator of subspace analysis (Section 3.2.1) is calculated from Eq. 3.39, that is,

$$\mathbf{H} = \mathbf{R}_x(\mathbf{R}_x + \sigma_v^2\mathbf{I})^{-1}$$

   where $\mathbf{R}_x$ is the autocorrelation matrix of the desired signal and $\sigma_v^2$ is the noise energy. Since $\mathbf{R}_x$ is not known, it must be estimated by some means. When we have the autocorrelation of the measured signal $\mathbf{R}_y$ and an estimate $\sigma_v^2$ of noise energy, we can then estimate $\mathbf{R}_x = \mathbf{R}_y - \sigma_v^2$.

   What can you say about the numerical condition of the LMMSE estimator? When is the inversion $(\mathbf{R}_x + \sigma_v^2)^{-1}$ ill-conditioned?

   (3 points)

4. The GSM standard applies two different regularisation methods to the autocorrelation sequence $r_k$ that is used for linear prediction. Namely, it applies lag-windowing $\widehat{r}_k = d_k r_k$ where $d_k = \alpha^{|k|}$ and $\alpha = 0.99$, and moreover, it adds a small coefficient $\mu = 10^{-6}$ to $\widehat{r}_0$, that is, $\tilde{\mathbf{R}} = \widehat{\mathbf{R}} + \mu\mathbf{I}$.

   Take a clean sound file from Noizeus and find a stable segment such as a vowel (for example, samples 7300 to 8200 of sp05.wav) and calculate its autocorrelation sequence of length 11 ($r_k$ with $0 \leq k \leq 10$).

(a) Study the conditioning of the autocorrelation matrix as a function of $\mu$ and $\alpha$. Specifically, plot the condition number as a function of $\mu \in [10^{-16}, 1]$ and $\alpha \in [0.8, 1]$.

(b) Study the spectrum of the autocorrelation for different values of $\mu$ and $\alpha$. Plot a number of examples.

*Tip*: Plotting the spectrum of an autocorrelation sequence is not as easy as it seems. You can use the following code that calculates spectrum using the corresponding linear predictive model:

```
m=10; % autocorrelation length
r = xcorr(x,m);
R = toeplitz(r(1+m+(0:m)));
a = inv(R)*[1;zeros(m,1)];
A = fft(a,512);
plot(linspace(0,fs,512),20*log10(abs(1./A)));
```

Discuss the results. Can you determine a suitable compromise between distortion of the spectrum and conditioning of the autocorrelation matrix? That is, how can you choose values for $\mu$ and $\alpha$?

(4 points)