The effect of inharmonicity on pitch in string instrument sounds

Hanna Järveläinen
Hanna.Jarvelainen@hut.fi

Tony Verma
Tony.Verma@acoustics.hut.fi

Vesa Välimäki
Vesa.Valimaki@hut.fi

Helsinki University of Technology
Laboratory of Acoustics and Audio Signal Processing
P.O.Box 3000, FIN-02015 HUT, Finland
http://www.acoustics.hut.fi

ABSTRACT

The effect of inharmonicity on pitch was measured by listening tests at four fundamental frequencies. Inharmonicity was defined in a way typical of string instruments, such as the piano, where all partials are elevated in a systematic way. It was found that the pitch judgment is usually dominated by some other partials than the fundamental; however, with a high degree of inharmonicity, the fundamental also began to dominate. Guidelines are given for compensating for the pitch difference between harmonic and inharmonic sounds in digital sound synthesis.

1. INTRODUCTION

It is well known that the stiffness of real strings causes wave dispersion by making the velocity dependent on frequency. If the string parameters are known, the inharmonic partial frequencies can be computed based on the wave equation. The partials are raised according to the following formula (Fletcher, Blackham & Stratton 1962):

$$f_n = nf_0\sqrt{1+Bn^2} \quad (1)$$

$$B = \frac{\pi^3 Q d^4}{64 l^2 T} \quad (2)$$

where n is the partial number, Q is Young's modulus, d is the diameter, l is the length and T is the tension of the string, and f_0 is the fundamental frequency of the string without stiffness. B is the inharmonicity coefficient for an unwrapped string. The value depends on the type of string and the string parameters. Completely harmonic partial frequencies are obtained with $B = 0$.

One of the most evident effects of inharmonicity is the stretched tuning of the piano to maintain harmonic consonance between musical intervals. Because of inharmonicity, the higher partials of low tones become sharp with respect to corresponding higher tones, and unpleasant beats occur. To minimize the beats, the bass range is tuned slightly flat and the treble range slightly sharp compared to the equal temperament. In the middle range, the adjustment is only a few cents (1/100 of a semitone), but at both ends of the keyboard, differences of 80-120 cents are common (Martin & Ward 1961). Lattard (1993) has simulated the stretched tuning process computationally.

Moore, Glasberg & Peters (1985) showed how mistuning a single partial affects the pitch of the whole complex. With mistuning up to about 2-3%, the residue pitch of the complex changes according to the amount of mistuning. With greater mistuning, the effect gets weaker, and finally the mistuned component segregates from the complex. Moore and his colleagues measured the change of the residue pitch due to mistuning of one partial and used it to approximate the relative dominance of the partials. They found that the pitch is built up as a weighted average of the lowest five or six harmonics. The weights were assigned to each harmonic according to the pitch effect that it caused. Later on, Dai (2000) suggested that the dominance region of pitch is not only connected to the partial number but also to absolute frequency, the most dominant partials being the ones nearest to 600 Hz.

There are few studies on sounds that exhibit systematic inharmonicity like string instruments. Slaymaker (1970) commented that the timbre of sounds with compressed or stretched octave relations is bell- or chimelike, and that chords played with such tones begin to sound out-of-tune. Mathews & Pierce (1980) found that stretching the partials affects tonal harmony by destroying the consonance of musically consonant chords. Thus the finality of cadences is reduced, for instance.

In digital sound synthesis, it is occasionally necessary to implement the effect of inharmonicity, while in other situations it might be neglected to achieve computational savings (Järveläinen, Välimäki & Karjalainen 1999). It is therefore expected that some finetuning is needed in order to maintain the correct musical scale between harmonic and inharmonic synthetic tones.

In the present study, the pitch change due to inharmonicity was measured as a function of the inharmonicity coefficient B with fundamental frequency as a parameter. Guidelines are given to approximate the pitch correction to compensate for the pitch increase in digital sound synthesis.

2. LISTENING TESTS

The pitch of inharmonic sounds was measured for four notes, A_1, G_3, A_4, and $C\sharp_6$. The task was to match the pitch of an inharmonic tone to that of a harmonic tone. The pitch increase was measured as a function of B for each note.

2.1. Test sounds

In order to generate realistic sounding tones for the experiment, sinusoidal modeling was used. Real piano tones were analyzed for both their inharmonic partials and soundboard response. To achieve this, the partials were modeled and subtracted from the original signal. This produced a residual signal that consisted of the characteristic 'knock' that occurs during the onset of a piano tone. By varying the frequency of each partial according to the inharmonicity cofficient B then adding in the pitchless 'knock', realistic sounding piano tones with varying degrees of inharmonicity were generated for the experiment. The original piano tones were taken from the Mcgill University Masters Series recordings (`http://www.music.mcgill.ca/resources/mums/html/mRecTech.htm`) and down sampled to 22.05 kHz.

Seven inharmonic test tones were generated for each note. The first test tone was completely harmonic with $B = 0$. The value of B increased fairly logarithmically to B_{max}, which was chosen for each note based on preliminary listening. With $B = B_{max}$, the pitch judgment was still possible but usually ambiguous. Each test tone had a duration of 0.9 seconds and consisted of the lowest six partials with amplitudes corresponding to the analyzed piano sounds. Six partials produce the correct pitch (Moore et al. 1985), and no more partials could even be used for $C\sharp_6$ before meeting the Nyquist limit. To study the possible effect of higher partials, corresponding test sounds with 12 partials were generated for A_1 and G_3. Table 1 summarizes the properties of the test sounds.

The adjustable tones were harmonic with $B = 0$. Sine tones were rejected for the practical reason that the aim is to correct the differences between harmonic and inharmonic tones, and because the pitch comparison was easier between sounds of similar timbre.

Note	f_0	B$_{max}$	Partials
A_1	55.0 Hz	0.005	6 or 12
G_3	196.0 Hz	0.01	6 or 12
A_4	440.0 Hz	0.01	6
$C\sharp_6$	1108.7 Hz	0.1	6

Table 1: Properties of the test sounds.

2.2. Subjects and test method

Six subjects participated the listening test. The listeners were personnel of the HUT Acoustics Laboratory, and most of them had a musical background as well as earlier experience of psychoacoustic tests. Two of the listeners were the authors HJ and TV. None of the subjects reported any hearing defects. The sounds were played through headphones in a silent listening room. The subjects were allowed to practise before the test.

The method of adjustment was used in the experiment. The task was to adjust the pitch of a harmonic tone until it matched that of an inharmonic tone. The fundamental frequency of the harmonic tone was changed by a scroll bar in the graphical user interface. The quantizing intervals were 0.2 Hz, 0.3 Hz, 0.4 Hz, and 0.5 Hz for the notes A_1, G_3, A_4, and $C\sharp 6$, respectively.

The test results were analyzed by the Analysis of Variance (ANOVA) for significant differences in the mean results for different B values. For further analysis, the Tukey HSD (Honestly Significant Difference) (Lehman 1991) was calculated in each case. It is a tool for pairwise follow-up tests, giving the smallest significant difference between pairs of means.

3. TEST RESULTS

The test results are shown in Fig. 1 for sounds with six partials. The results are consistent when B is reasonably small, but with the highest values two different tendencies can be seen. Some of the subjects apparently chose the pitch of the fundamental that segregated from the complex, while others were still judging according to the overall impression.

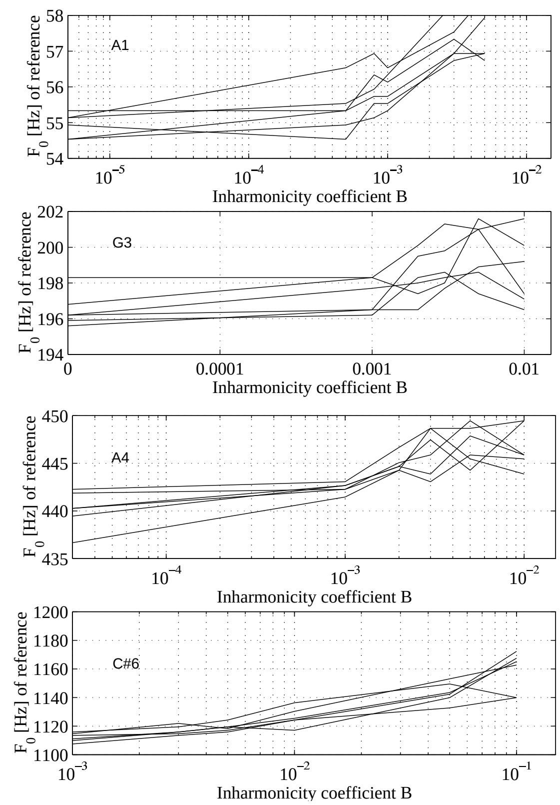

Figure 1: The individual results for A_1, G_3, A_4, and $C\sharp_6$ with 6 partials (solid lines), and corresponding mean results (dashed lines).

The ANOVA gave highly significant results for all notes, suggesting that there is a true difference in perceived pitch with at least one B value for each note. However, with small B values the pitch changes were statistically insignificant, as was seen by the Tukey HSD test. The test results obtained by using 12 or 6 partials did not differ significantly.

It was found that the pitch was dominated by some higher partial than the fundamental, as seen in Fig 2. For A_1, the mean pitch contour followed the sixth partial. The pitch was judged equal to the frequency of the sixth partial divided by six. With increasing fundamental frequency, the number of the dominating partial came down. For G_3 and A_4, the third partial was dominant, and the second one dominated for $C\sharp_6$.

The dominant partial pitch estimate as well as the fundamental frequency of the test tone are shown by dashed lines. The mean pitch judgments are shown by solid lines. It is seen that the judged pitch separates from the estimate at the highest values of B and drops towards the fundamental. In this area the pitch judgment was very hard, since individual partials started to segregate from the complex.

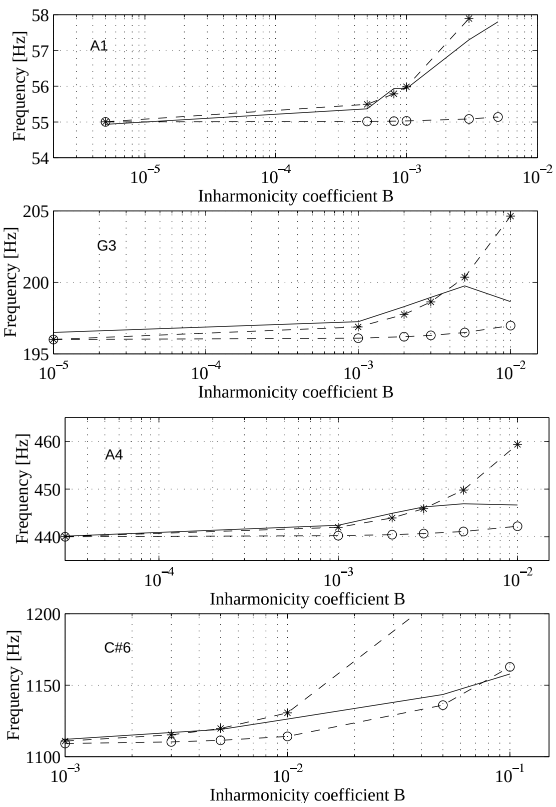

Figure 2: Estimation of the pitch according to the dominant partial. Mean of the pitch judgments (solid lines), fundamental frequency (o), and the pitch estimate (*). The pitch estimates top to bottom: $(f_6/6)$, $(f_3/3)$, $(f_3/3)$, and $(f_2/2)$.

The relevant range of the dominant partial pitch estimate as a function of B was approximated statistically by using the Tukey HSD measure. The lower limit was the B value where the judged pitch had increased at least by one HSD, and the upper limit was where the judged pitch was one HSD lower than the estimate. Below the lower limit there would be no significant pitch increase, and above the upper limit the pitch judgment would be ambiguous. The limits are summarized in Table 2.

One of the main objectives of this study was to find out the relation of pitch and timbre effects of inharmonicity. In an earlier study, pitch corrections were needed when the timbral effect was studied. To compare the audibility of pitch and timbre effects, a straight line fit was made to model the lower boundary of significant pitch increase B_{lower} as a function of fundamental frequency. The result was

Note	Tukey HSD	B_{lower}	B_{upper}
A_1	1.19 Hz	0.0012	0.0040
G_3	2.46 Hz	0.0029	0.0071
A_4	3.17 HZ	0.0013	0.0052
$C\sharp_6$	12.6 Hz	0.0069	0.0153

Table 2: The data analysis results for the studied notes. The Tukey Honestly significant difference, and the relevant range of the dominant partial pitch estimation (B_{lower} and B_{upper}).

$$\ln(B_{lower}) = 0.459\ln(f) - 8.62 \quad (3)$$

For the detection threshold of differences in timbre the corresponding straight line fit was made in (Järveläinen et al. 1999):

$$\ln(B_{timbre}) = 2.54\ln(f) - 24.6 \quad (4)$$

The lines are shown together in the upper plot of Fig. 3. It can be seen that at low fundamental frequencies, the difference in timbre is perceived at much lower values of B than the pitch difference. At higher fundamental frequencies, the lines approach each other, and the slopes suggest that they might even cross at a still higher frequency. This would mean that at fundamental frequencies greater than 2000 Hz, the pitch increase should be compensated even if the timbral differences could be ignored.

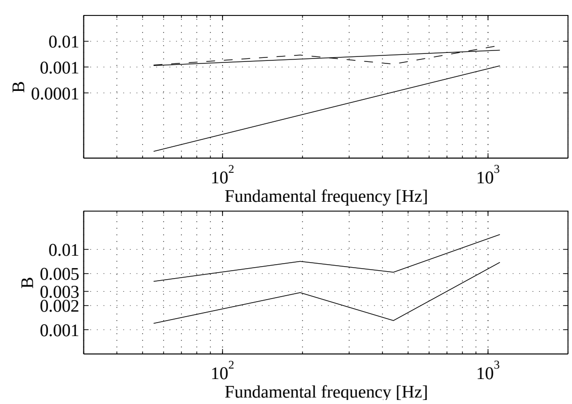

Figure 3: Top: Straight line fits to the limit of significant pitch difference B_{lower} (upper line) and the detection threshold of timbral effects (lower line). Bottom: The relevant range of dominant partial pitch estimation (B_{lower} and B_{upper}).

4. DISCUSSION

Pitch compensation might be necessary in string instrument synthesis if inharmonicity is partly ignored. Another field of application is the synthesis of very inharmonic sounds. In both cases, corrections are needed to maintain the musical scale.

Typical values of B for piano strings lie roughly between 0.00005 for low bass tones and 0.015 for the high treble tones (Conklin 1999). In the bass range, pitch corrections are surely unnecessary, even though it would be needed to implement the effect of inharmonicity on timbre. In the highest treble, the situation might be the opposite. The results suggest that the threshold for detecting timbral differences is very close to the threshold of detecting pitch differences at high fundamental frequencies. It is possible that the thresholds cross each other at approximately 2000 Hz. In that case the pitch effect should be compensated for even if the timbral effect was inaudible.

According to the current results, compensating for the pitch differences between harmonic and inharmonic synthetic tones is not too complicated. We have presented lower boundaries for the B parameter, below which no significant pitch effect was observed, and shown how the average pitch judgments follow some of the higher partials. More accurate models are needed to cover the entire pitch range of the piano, however.

Our results are consistent with the previous research that was discussed in the introduction. In the work of Moore et al. (1985), Dai (2000), and the pitch models of Terhardt, Stoll & Seewann (1982) and Goldstein (1973), it is stated that the pitch percept is based on only the lowest partials. This is also seen from our tests with 6 and 12 partials. The results were similar regardless of the number of harmonics. However, previous research disagrees on the dominance of individual partials. Our results with systematically mistuned sounds are nearest to those of Dai (2000), who suggested that the dominant partials are around 600 Hz in frequency. We found that the dominant partial number decreased with increasing fundamental frequency. At the lowest fundamental, the pitch judgment followed an estimate given by the sixth partial, while at the highest fundamental, the second partial was dominant. However, we found no absolute dominance of the 600 Hz region.

The pitch judgment was strongly dominated by only one partial, until at high values of B the fundamental became dominant as well. The earlier models of dominance (Moore et al. (1985) and Terhardt et al. (1982)) state that the pitch is a weighted average of several components. This suggests some further research.

It also remains a future task to find out to what degree the pitch effect and the timbral effect are independent of each other. While the pitch only depends on the lowest partials, the timbral effect apparently strengthens as the number of partials increases. The detection of a change in timbre seems to be related to the higher unresolvable partials, but surely the lower partials have an effect as well.

5. ACKNOWLEDGMENTS

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