Interpolated and Warped 2-D Digital Waveguide Mesh Algorithms

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Outline

- Introduction
- 2-D Digital Waveguide Mesh Algorithms
- Frequency Warping Techniques
- Extending the Frequency Range
- Numerical Examples
- Conclusions

Introduction

- Digital waveguides for physical modeling of musical instruments and other acoustic systems (Smith, 1992)
- 2-D digital waveguide mesh (WGM) for simulation of membranes, drums etc. (Van Duyne & Smith, 1993)
- 3-D digital waveguide mesh for simulation of acoustic spaces (Savioja *et al.*, 1994)
  - Violin body (Huang *et al.*, 2000)
  - Drums (Aird *et al.*, 2000)

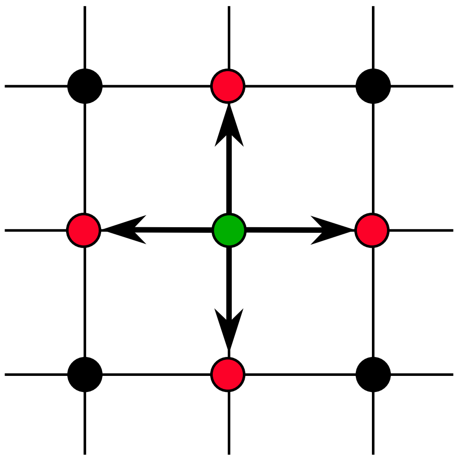
Sophisticated 2-D Waveguide Structures

- In the original WGM, wave propagation speed depends on direction and frequency (Van Duyne & Smith, 1993)
- More advanced structures ease this problem, e.g.,
  - Triangular WGM (Fontana & Rocchesso, 1995, 1998; Van Duyne & Smith, 1995, 1996)
  - Interpolated rectangular WGM (Savioja & Välimäki, ICASSP'97, IEEE Trans. SAP 2000)
- Direction-dependence is reduced but frequency-dependence remains

⇒ Dispersion

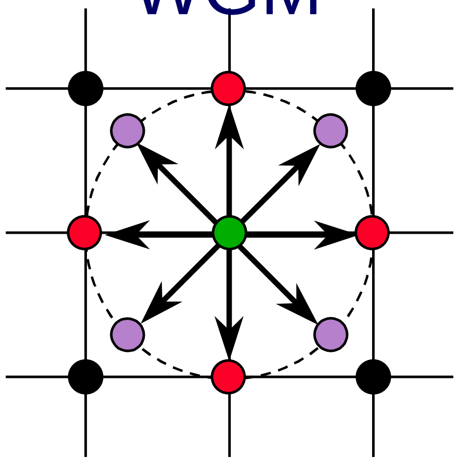
Interpolated Rectangular Waveguide Mesh

Original WGM

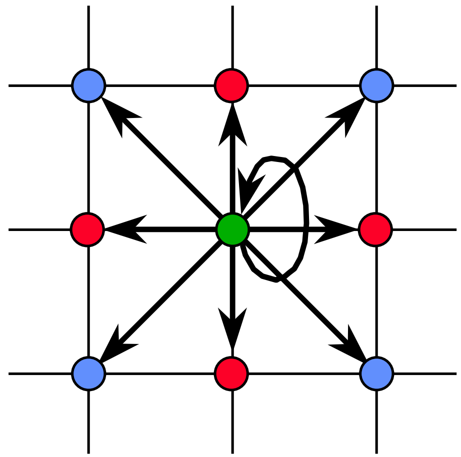


(Van Duyne & Smith, 1993)

Hypothetical 8-directional WGM



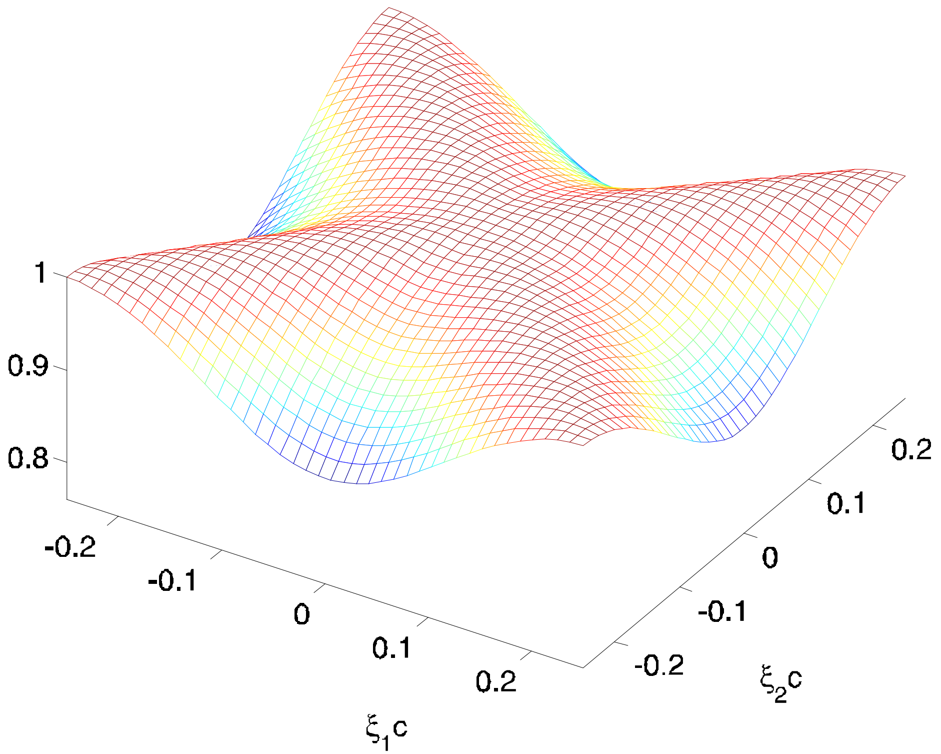
Interpolated WGM



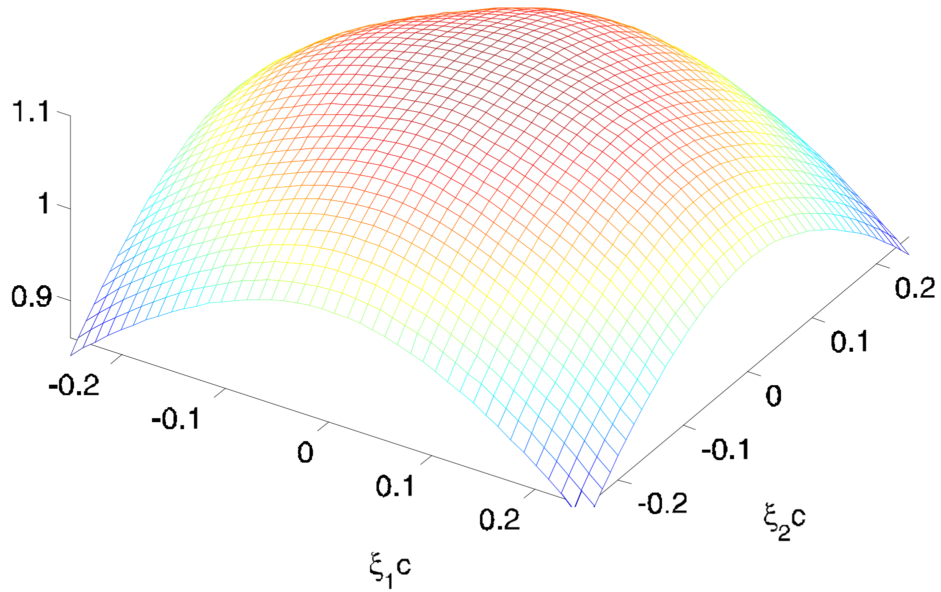
(Savioja & Välimäki, 1997, 2000)

Wave Propagation Speed

Original WGM

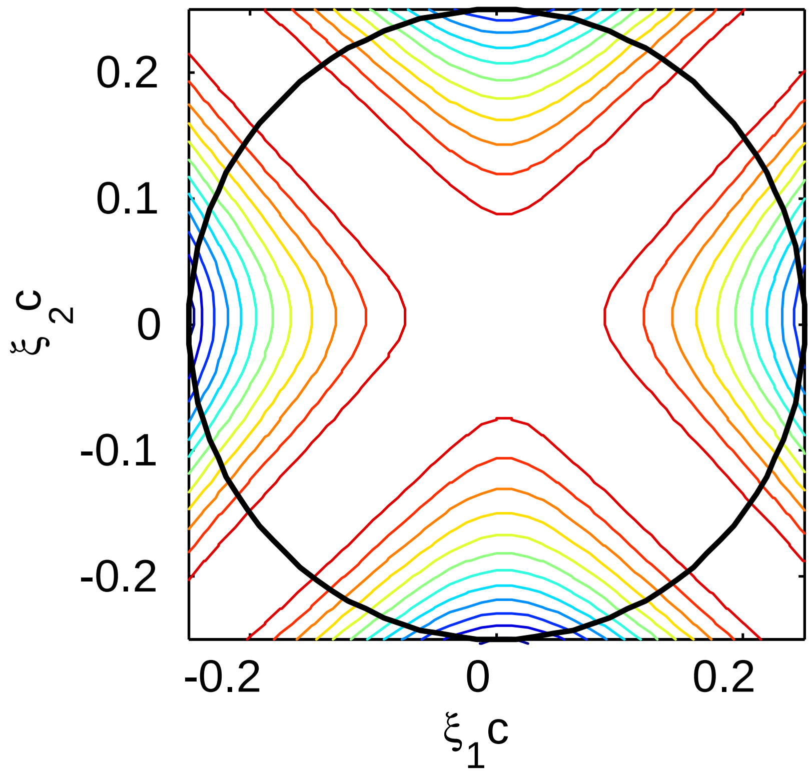


Interpolated WGM
(Bilinear interpolation)

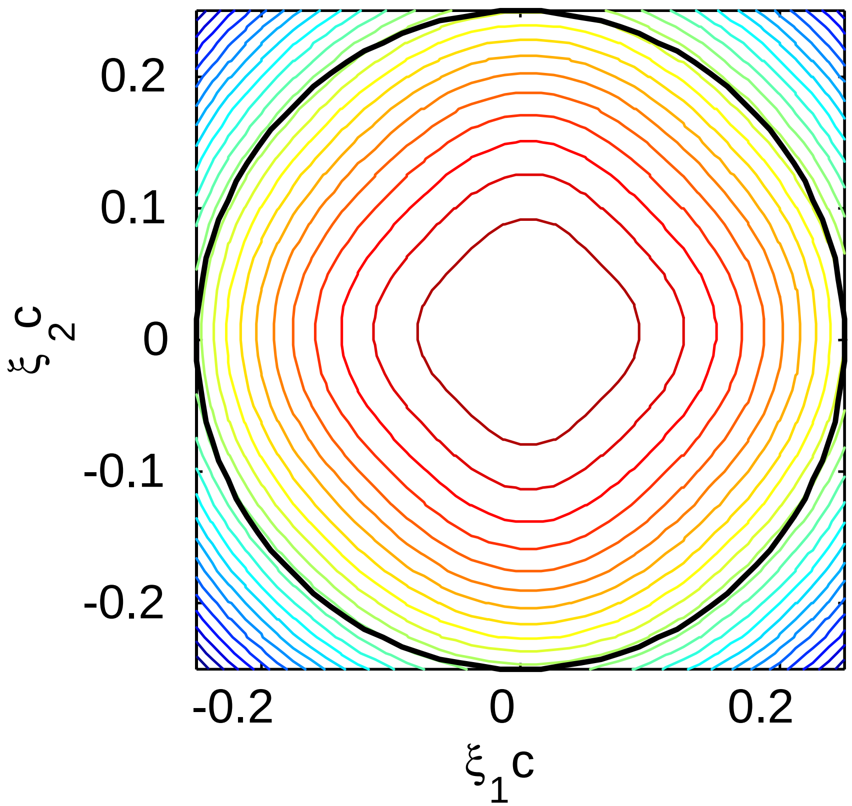


Wave Propagation Speed (2)

Original WGM

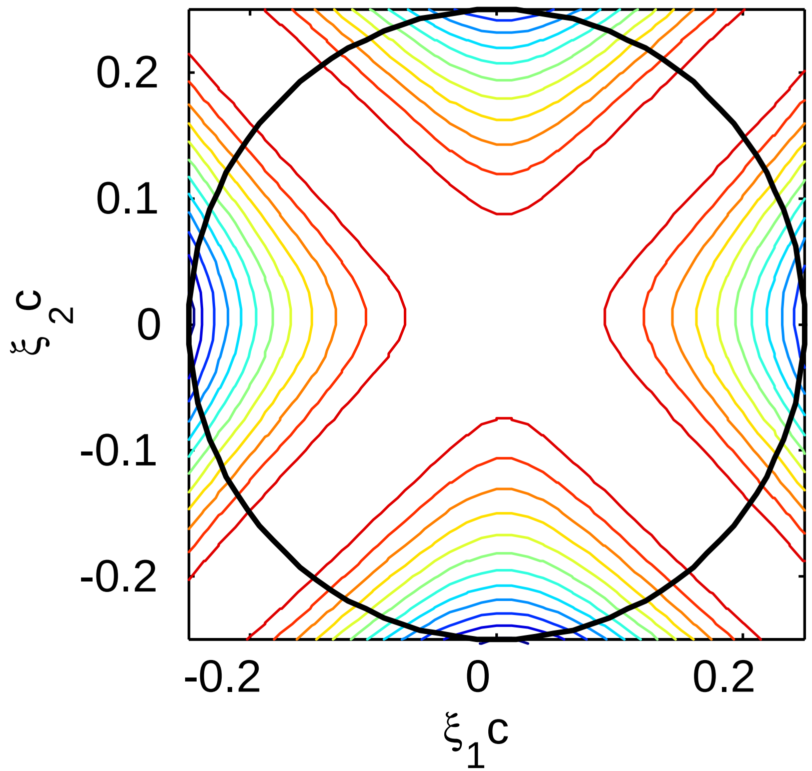


Interpolated WGM (Bilinear interpolation)

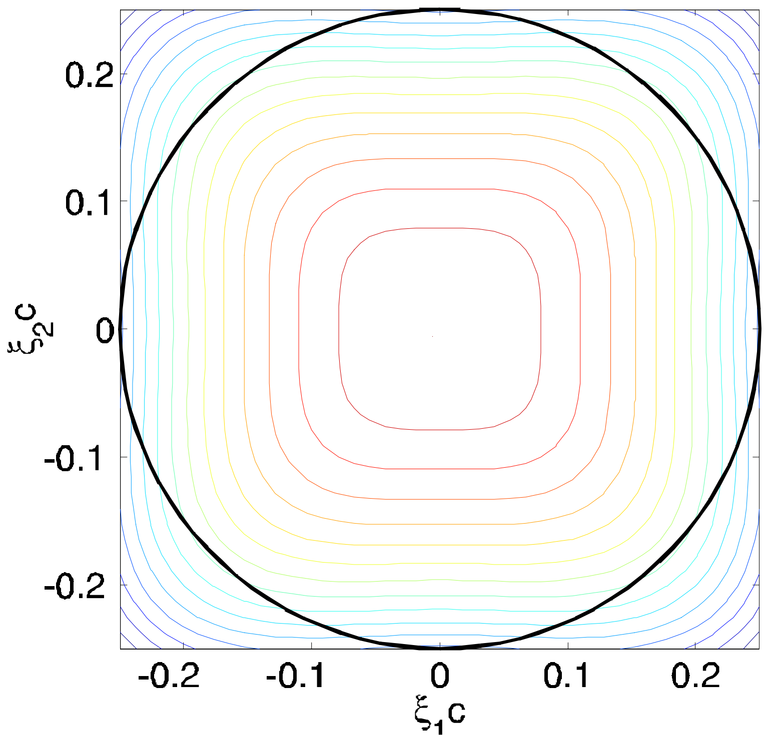


Wave Propagation Speed (3)

Original WGM

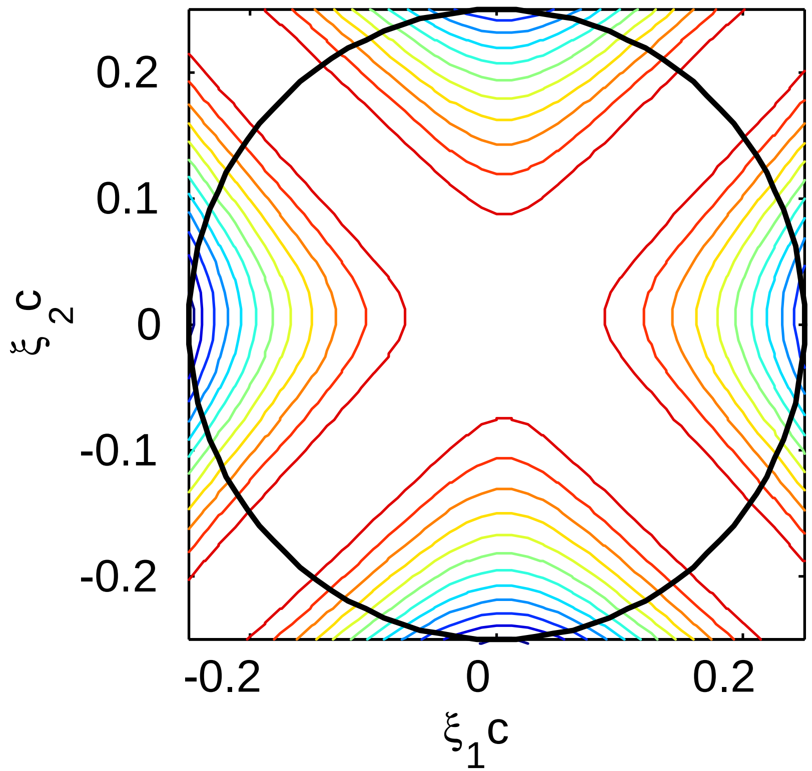


Interpolated WGM (Quadratic interpolation)

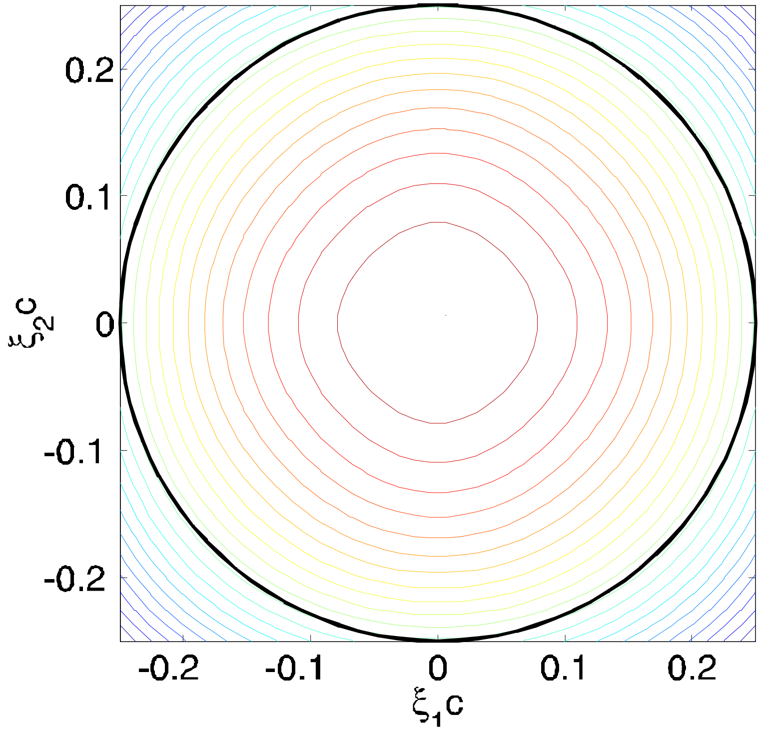


Wave Propagation Speed (4)

Original WGM



Interpolated WGM
(Optimal interpolation)



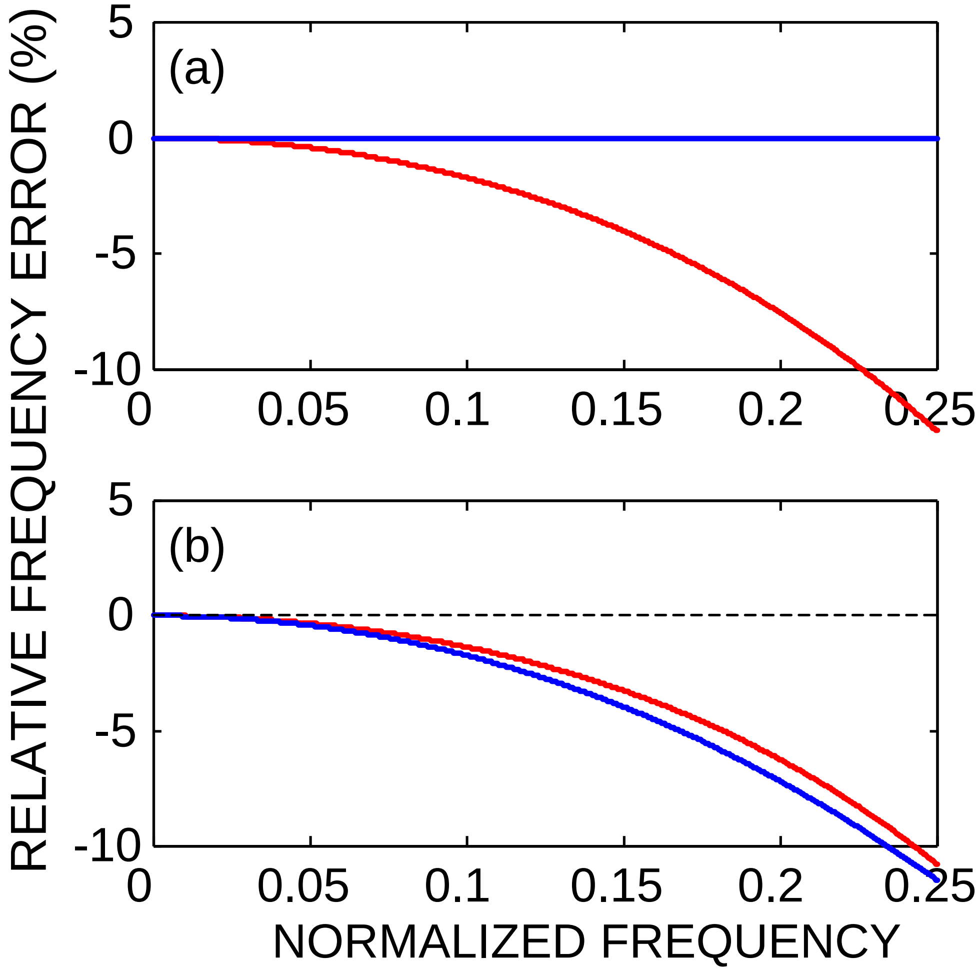
(Savioja & Välimäki, 2000)

Relative Frequency Error (RFE)

RFE in diagonal and axial directions:

(a) original and

(b) bilinearly interpolated rectangular WGM

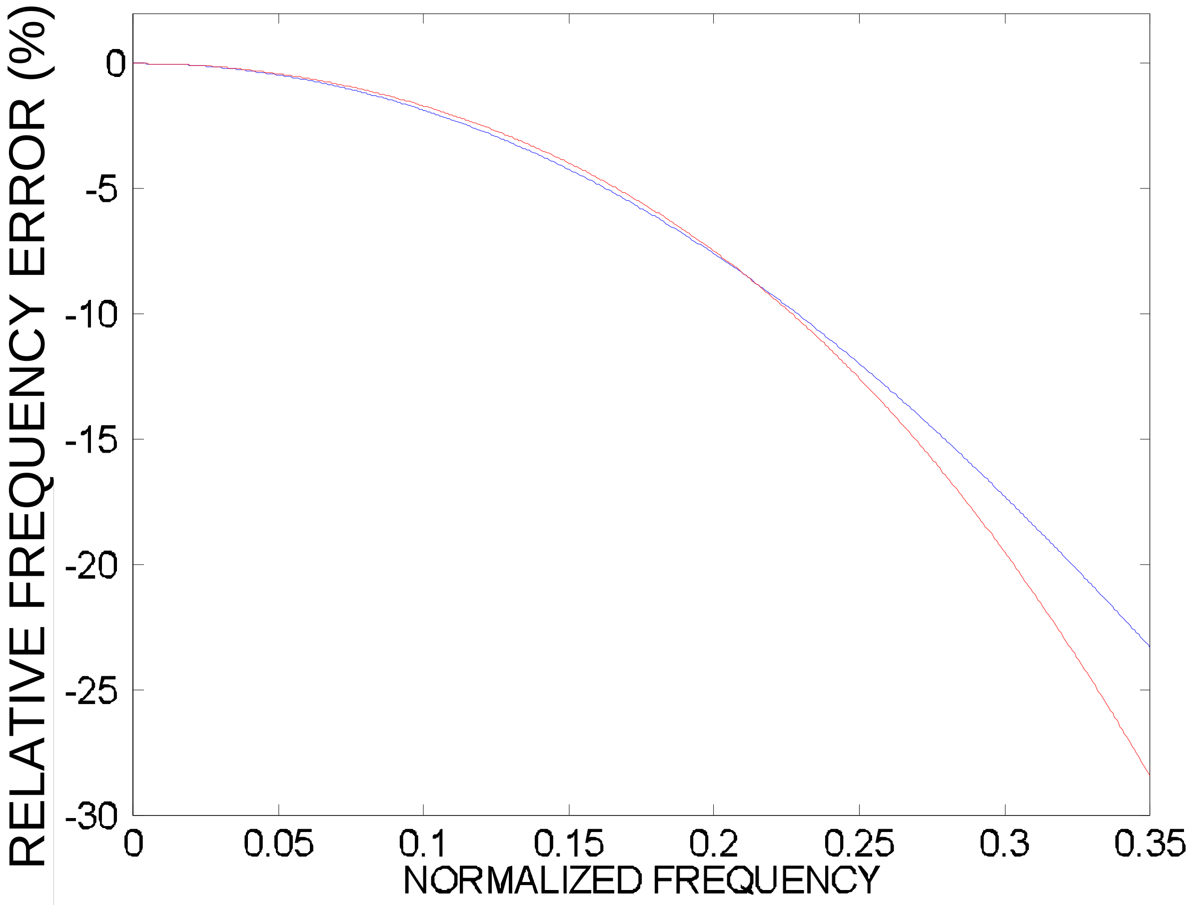


Relative Frequency Error (RFE) (2)

RFE in diagonal and axial directions:

Optimally interpolated rectangular WG mesh

(up to 0.25f_s)



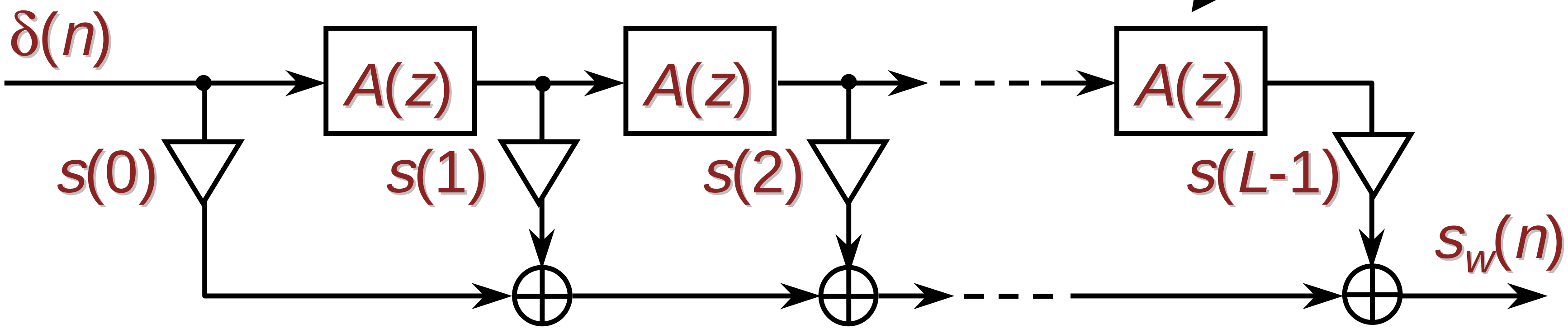
Frequency Warping

- Dispersion error of the interpolated WGM can be reduced using frequency warping because
  - The difference between the max and min errors is small
  - The RFE curve is smooth
- Postprocess the response of the WGM using a warped-FIR filter (Oppenheim *et al.*, 1971; Härmä *et al.*, JAES, Nov. 2000)

Frequency Warping: Warped-FIR Filter

- Chain of first-order allpass filters

$$A(z) = \frac{z^{-1} + \lambda}{1 + \lambda z^{-1}}$$



- $s(n)$ is the signal to be warped
- $s_w(n)$ is the warped signal
- The extent of warping is determined by λ

Optimization of Warping Factor λ

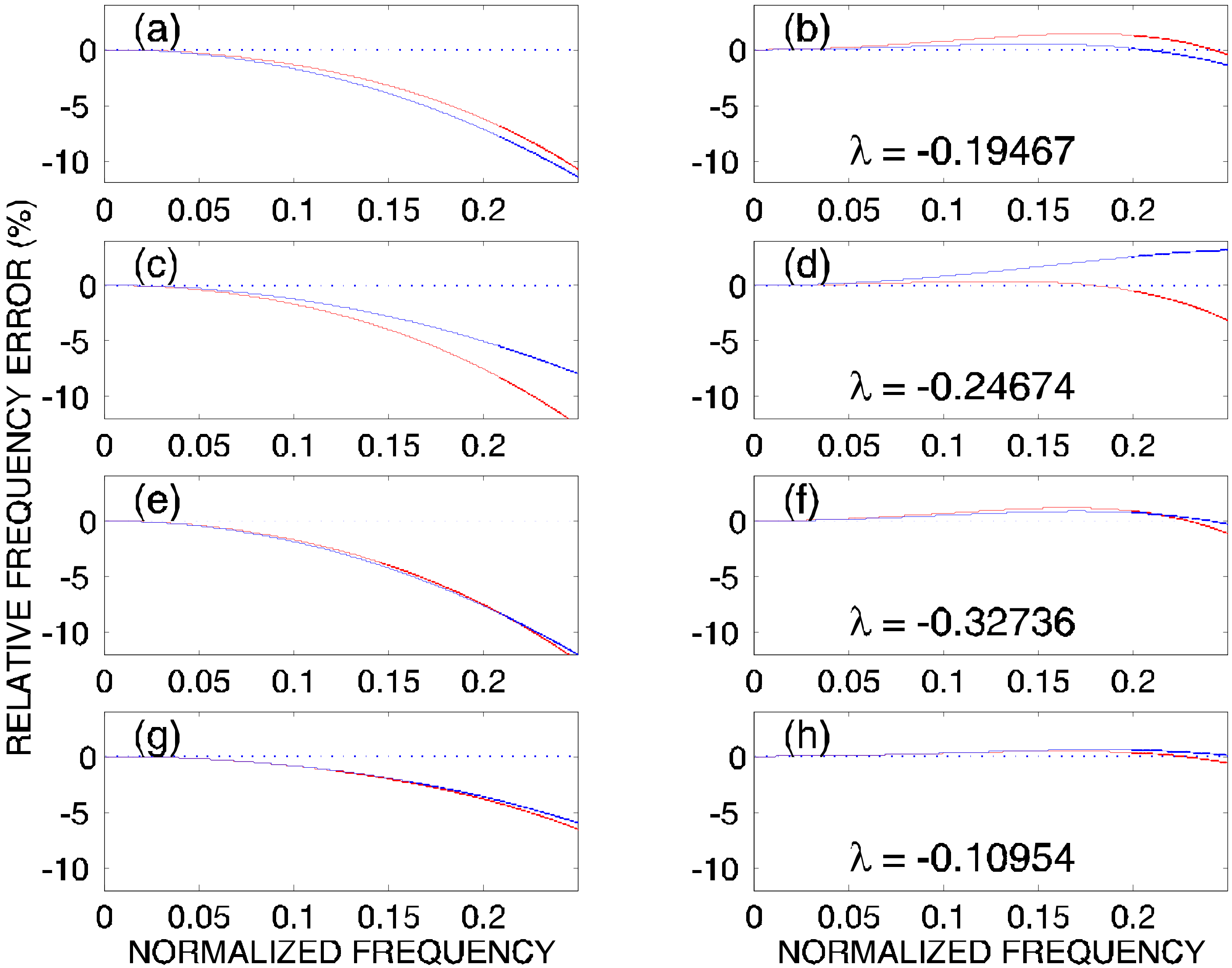
- Different optimization strategies can be used, such as
  - least squares
  - minimize maximal error (minimax)
  - maximize the bandwidth of X% error tolerance
- We present results for minimax optimization

(a,b) Bilinear interpolation

(c,d) Quadratic interpolation

(e,f) Optimal interpolation

(g,h) Triangular mesh



Välimäki and Savioja 2000

Higher-Order Frequency Warping?

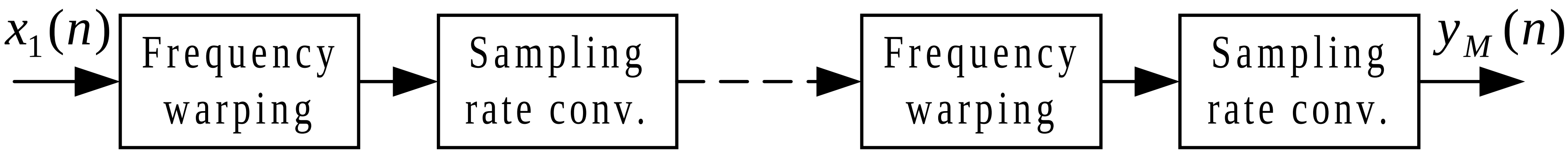
- How to add degrees of freedom to the warping to improve the accuracy?
  - Use a chain of higher-order allpass filters?

    Perhaps, but aliasing will occur... ***No****.*
  - Many 1st-order warpings in cascade?

    No*, because it's equivalent to a single warping using* $(\lambda_1 + \lambda_2) / (1 + \lambda_1\lambda_2)$
- There is a way...

Multiwarping

- Every frequency warping operation must be accompanied by sampling rate conversion
  - All frequencies are shifted by warping, including those that should not
- Frequency-warping and sampling-rate-conversion operations can be cascaded
  - Many parameters to optimize: λ_1, λ_2, ... D_1, D_2,...



Reduced Relative Frequency Error

(a) Warping with

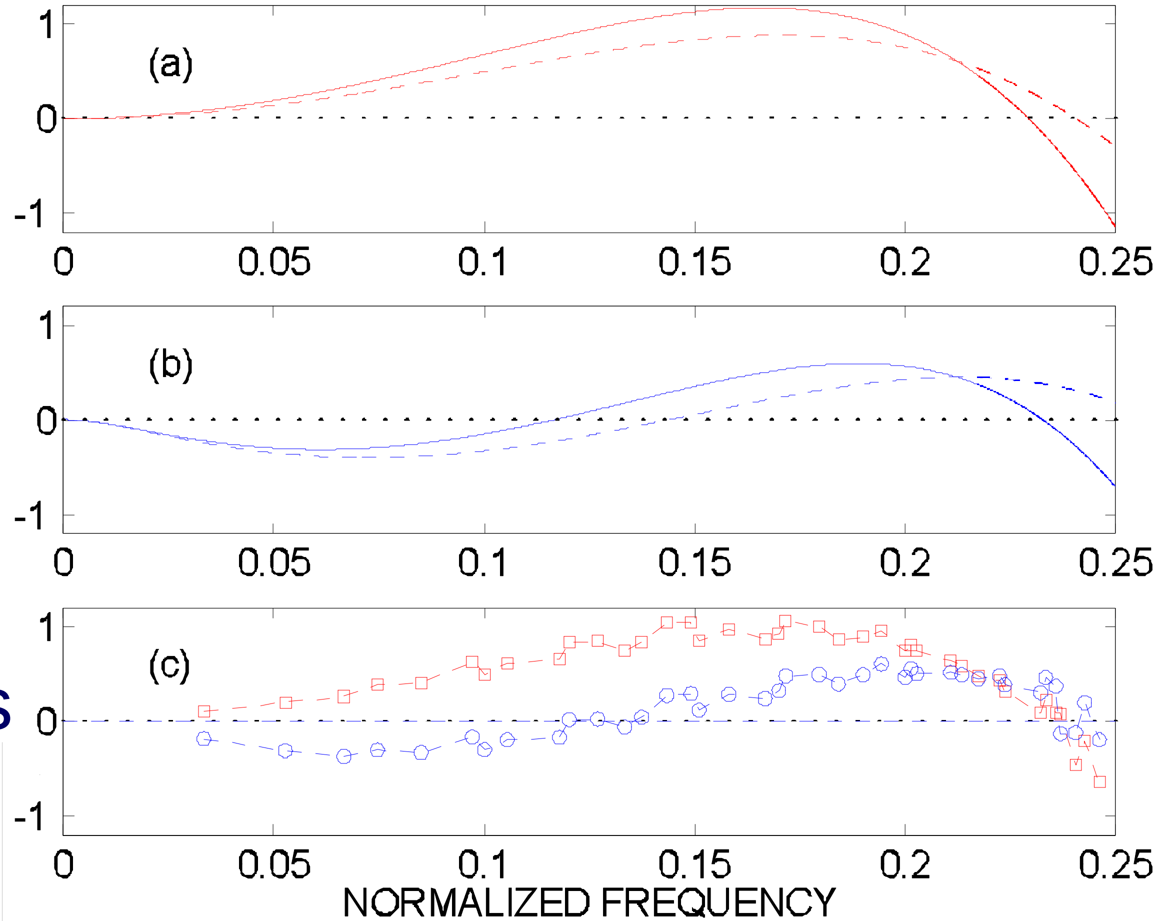
$\lambda = -0.32$

(b) Multiwarping with

$\lambda_1 = -0.92,\ D_1 = 0.998$

$\lambda_2 = -0.99,\ D_2 = 7.3$

(c) Error in eigenmodes



Computational complexity

- Original WGM: 1 binary shift & 4 additions
- Interpolated WGM: 3 MUL & 9 ADD
- Warped-FIR filter: $O(L^2)$ where L is the signal length
- Advantages of interpolation & warping
  - Wider bandwidth with small error: up to 0.25 instead of 0.1 or so
  - If no need to extend bandwidth, smaller mesh size may be used

Extending the Frequency Range

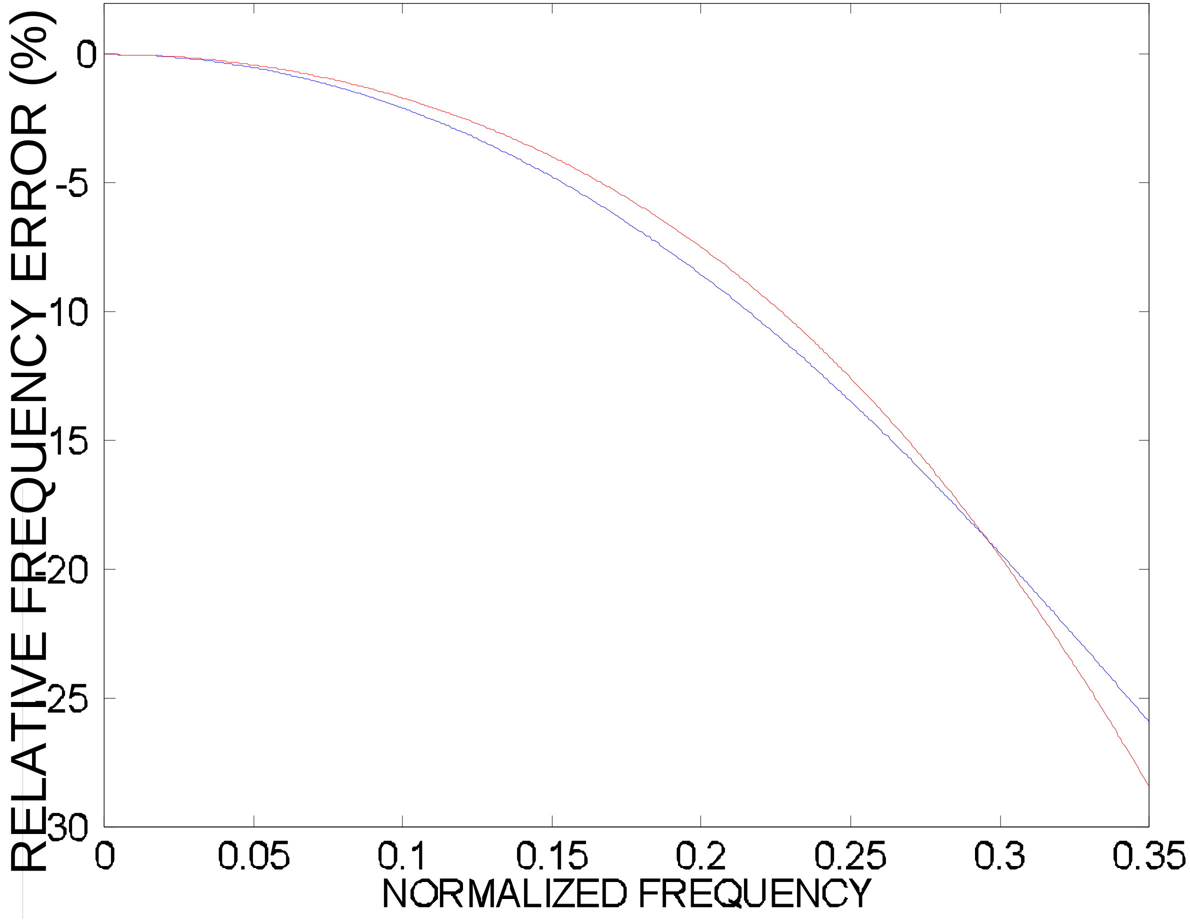
- It is known that the limiting frequency of the original waveguide mesh is 0.25
  - The point-to-point transfer functions on the mesh are functions of z^{-2} , i.e., oversampling by 2
- Fontana and Rocchesso (1998): triangular WG mesh has a wider frequency range, up to about 0.3
- How about the interpolated WG mesh?
  - The interpolation changes everything
  - Maybe also the upper frequency changes...

Relative Frequency Error (RFE) (2)

RFE in diagonal and axial directions:

Optimally interpolated rectangular WG mesh

(up to 0.35f_s)



Extending the Frequency Range (3)

- The mapping of frequencies for various WGMs

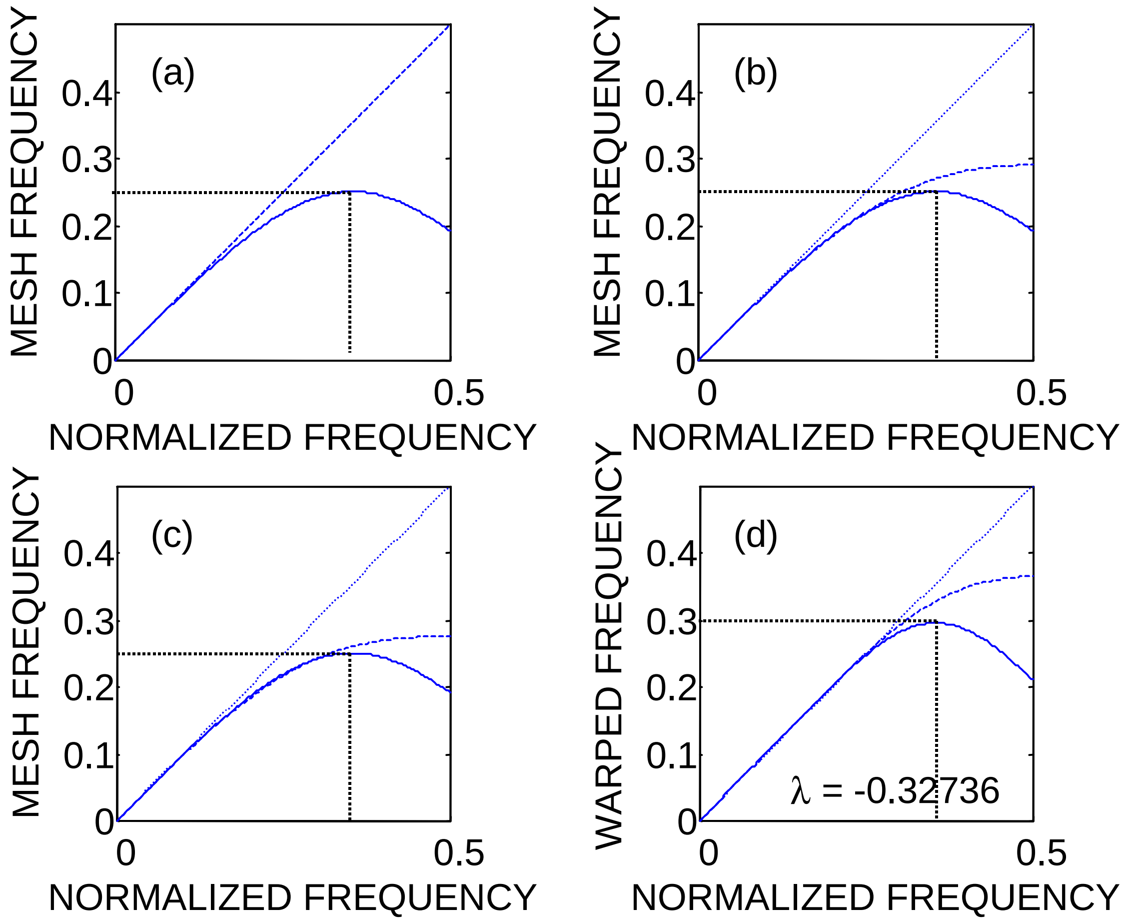
Upper frequency limit always **0.3536**

(a) Original

(b) Optimally interp. up to 0.25

(c) Optimally interp. up to 0.35

(d) Warped case b



Simulation Result vs. Analytical Solution

Magnitude spectrum of a square membrane

(a) original

(b) warped interpolated

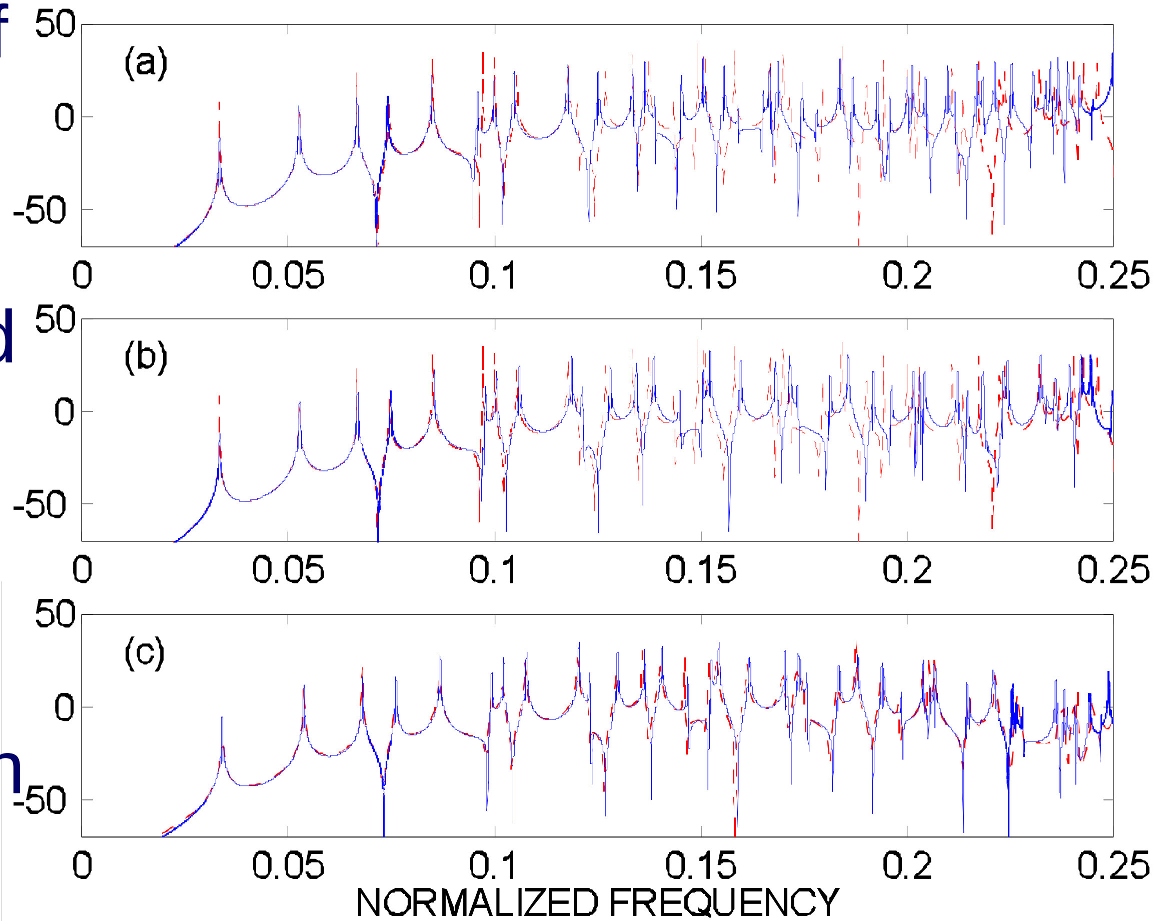
($\lambda = -0.32736$)

(c) warped triangular

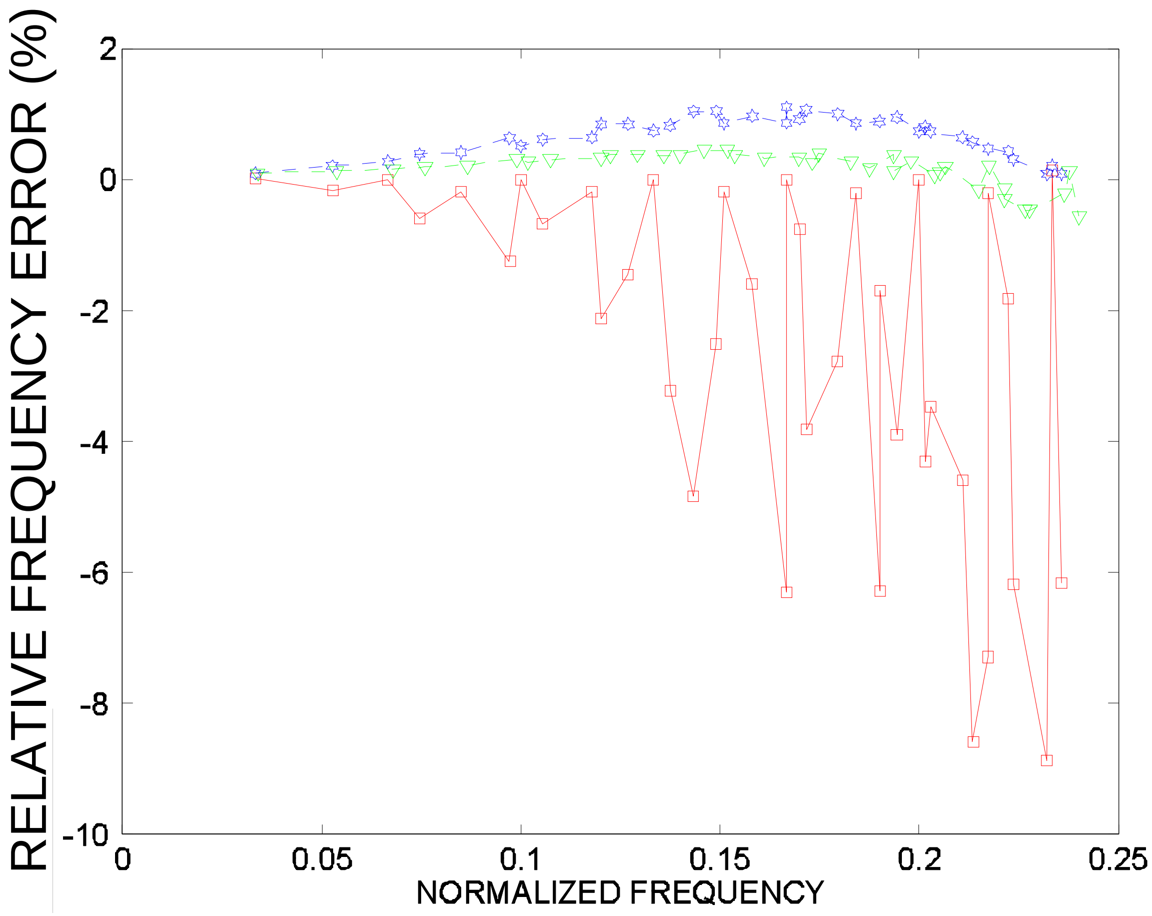
($\lambda = -0.10954$)

digital waveguide mesh

(with ideal response in the background)



Error in Mode Frequencies



Error in eigenfrequencies of a square membrane

- Warped interpolated WGM
- Warped triangular WGM
- Original WGM

Conclusions and Future Work

- Accuracy of 2-D digital waveguide mesh simulations can be improved using
  1) the interpolated or triangular WGM and
  2) frequency warping or multiwarping
- Dispersion can be reduced dramatically
- In the future, the interpolation and warping techniques will be applied to 3-D WGM simulations
- Modeling of boundary conditions and losses must be improved