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SIGNAL-DEPENDENT NONLINEARITIES FOR PHYSICAL MODELS USING TIME-VARYING FRACTIONAL DELAY FILTERS

Vesa Välimäki, Tero Tolonen, and Matti Karjalainen
Helsinki University of Technology, Laboratory of Acoustics and Audio Signal Processing, Espoo, Finland
vesa.valimaki@hut.fi, tero.tolonen@hut.fi, matti.karjalainen@hut.fi
http://www.acoustics.hut.fi/

Abstract

This paper discusses the use of nonlinear filters that are based on linear digital filters with input-signal-dependent coefficients in the context of digital waveguide synthesis models. It is proposed that the practical uses of such filters in music synthesis may be generalized to a time-varying fractional delay (TVFD) filter with signal-dependent coefficients. Three different cases are outlined: 1) simulation of a pressure-dependent wave propagation speed in a wind instrument bore, 2) nonlinear termination of a vibrating string, and 3) a tension-dependent variable speed of transverse vibrations on a string. Implementation structures for TVFD filters based on FIR filters are suggested.

1 Introduction

In physical modeling of musical instruments, nonlinearities present in real instruments have typically been discarded whenever possible, and computationally tractable and efficient linear signal processing algorithms have been devised for implementation. Recently, methods have been proposed for simulating *passive nonlinear phenomena* exhibited by a string or a plate (Pierce and Van Duyne 1997) and for modeling the nonlinear propagation of pressure waves in brass instruments (Msallam *et al.* 1997; Tassart *et al.* 1997). This paper generalizes the concept of a signal-dependent nonlinearity and suggests the implementation of such devices using *time-varying fractional delay* (TVFD) filters with signal-dependent coefficients. The proposed method is applicable to simulation of a wide range of nonlinear vibrations, including the vibration of a string attached to a longitudinally yielding bridge, signal-dependent tension of a vibrating string, and air column vibrations when the sound pressure is very high, e.g., in brass instruments. This paper suggests the use of FIR fractional delay filters for realizing signal-dependent passive nonlinearities.

Figure 1 illustrates a general nonlinearity that consists of a TVFD filter with *feedforward delay control* using the input signal. The block labeled by G converts the input signal sample values into a control parameter $d(n)$, the real-valued delay parameter. The TVFD filter may be one of the formerly proposed fractional delay filters (Laakso *et al.* 1997). Tassart *et al.* (1997) have proposed a similar structure with feedback control of the delay. In general, however, the TVFD filter is inside the feedback loop of a digital waveguide model, and so the output signal contributes to the input signal and to the delay. In Section 2, three different kinds of nonlinearities in musical instruments and their models are discussed. In Section 3, signal processing algorithms to implement signal-dependent nonlinearities are tackled. FIR FD filter structures are proposed for implementation of passive nonlinearities, and an efficient algorithm for simulation of a variable string tension is introduced. Section 4 concludes the paper.

2 Nonlinearities of Musical Vibrating Structures

In many previous works, musical instruments have been modeled according to the framework of McIntyre *et al.* (1983), i.e., as a nonlinear excitation element that is coupled to a linear resonator. The linear resonator model has produced good results with many instruments, such as the guitar and the flute. The assumption of linearity of the resonator is not feasible in all cases. In fact, the resonator models are typically derived from the linearized wave equations that assume small amplitude oscillations. Clearly, such a model cannot be expected to produce authentic results for simulations of large-amplitude vibration. Recently, several approaches to take into account the nonlinearities related to the behavior of the resonator have been proposed. Three different cases are discussed in the following.

1. The nonlinearity of the wave propagation exhibited by brass instruments is caused by propagation speed that

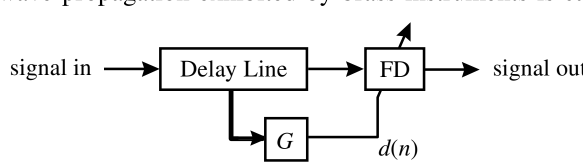

Figure 1: A general system where the input signal controls the delay parameter d(n) of a fractional delay (FD) filter. Function G computes d(n) based on the input signal sample(s).

varies depending on the amplitude of the wave (Msallam *et al.* 1997). The signal-dependent propagation speed can be formulated as a modulation on the length of the delay line in the digital waveguide simulation and implemented efficiently using fractional delay filters (Msallam *et al.* 1997; Tassart *et al.* 1997). Note that in this case the wave propagation is non-uniform in the bore resulting in distortion of the waveform. The simulation of the nonlinearity has been reported to yield realistic synthesis results (Msallam *et al.* 1997).

2. Pierce and Van Duyne (1997) have presented a model of a string that is terminated to a double-spring apparatus. The stiffness constants of the two springs are unequal, and the string is only attached to one of the springs at a time. At each crossing of the equilibrium position the string is detached from one spring and attached to the other. The discrete-time model is implemented with a first-order allpass filter whose coefficient is altered at each zero crossing of the state of the allpass filter. This nonlinear structure is approximately passive and energy-conserving.

3. The large-amplitude vibration in a string exhibits yet another type of nonlinearity. The primary source for the nonlinear behavior is tension that increases as the length of the string increases (Legge and Fletcher 1984). Since the speed of longitudinal waves is considerably larger than that of the transversal waves, tension is approximately uniformly distributed in the string, which results in modulated propagation speed. Karjalainen *et al*. (1993) have simulated the nonlinear coupling of the tension modulation to the output waveform of the kantele, a traditional Finnish plucked string instrument. In Section 3.2 of this paper we propose a method to implement the simulation of a wave with uniformly distributed signal-dependent propagation speed.

All the aforementioned nonlinearities can be formulated as signal-dependent time-varying implementations using fractional delay filters. This corresponds to modulation of the delay length in a digital waveguide model.

3 Implementation of Time-Varying Fractional Delay Filters

In practice, it is essential that the coefficients of the FD filter in Fig. 1 can be updated quickly, that is, with only a few operations. Two alternative implementation strategies are available: (1) tabulation of coefficient values for a possible range of parameters $d(n)$ and updating of coefficients with table lookup, or (2) the use of a special filter structure for the FD filter, where the structure itself contains parameter d. In the former case, any design method is feasible since the filter design is conducted before run-time. For the latter case, efficient FIR structures are suggested in Section 3.1.

Pierce and Van Duyne (1997) used a first-order allpass filter to generate nonlinear effects for synthetic string sounds. While time-varying recursive filters usually suffer from transient problems, their algorithm minimizes this effect by switching the filter coefficient at the zero crossing of the state variable of the allpass filter. Strictly speaking, in the case of discrete-time signals, zero crossings where the signal value is exactly zero do not exist in general. A transientless time-varying allpass filter can be implemented as suggested by Välimäki *et al.* (1995, 1998). In this case, the implementation structure consists of two allpass filters running in parallel. The output signal is obtained by selecting the output of one filter at a time, instead of changing the coefficient of one filter.

3.1 TVFD Element Based on FIR Filters

Let us consider the implementation of a time-varying fractional delay FIR filter. Figure 2(a) shows a particular implementation of the TVFD filter, which consists of two linear interpolators. The signal value modulates the fractional delay parameter $d(n)$, the value of which is limited by a constant scaling coefficient c and by taking the absolute value. The delay line is switched in Figure 2(a) based on the sign of the input signal $x(n)$, as proposed by Pierce and Van Duyne (1997). Thus the produced delay is $1 - d(n)$ or $1 + d(n)$ samples, which we call cases 1 and 2, respectively ($0 \le d(n) \le 1$). The output signal $y(n)$ of the filter is computed as $y(n) = d(n)x(n) + [1 - d(n)]x(n - 1)$ in case 1, and as $y(n) = [1 - d(n)]x(n - 1) + d(n)x(n - 2)$ in case 2. The computational complexity of the structure is the same as that of a first-order allpass filter (when implemented using one multiplier): 2 unit delays, 2 additions, and 1 multiplication plus operations required to implement mapping function G (multiplication by c and taking the absolute value in this case). An advantage of the FIR structure is that the delay can be changed every sample interval. No tran-

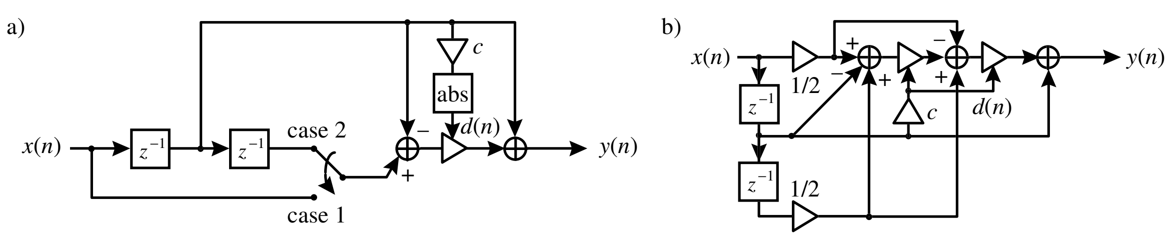

Figure 2: a) A realization structure for time-varying symmetric linear interpolator. b) A modified Farrow structure for second-order Lagrange interpolation with a signal-dependent delay parameter.

sients are generated due to coefficient changes—however, a discontinuity is observed in the output signal $y(n)$ at the time of change. For the definition and discussion on transients and discontinuities, see Välimäki and Laakso (1998).

A disadvantage of FIR FD filters is that they do not have a flat magnitude response (see Laakso *et al.* 1996). However, in practice the TVFD filter is inside a feedback loop, which typically also includes losses; the FIR filter automatically implements some of these losses. The symmetrical switching ensures that no amplitude modulation occurs, since the magnitude responses in cases 1 and 2 are identical. In practice, the variation of the filter coefficient may be very small to obtain a desired nonlinear effect and thus the amplitude modulation effect may be negligible.

Another choice of FIR-type TVFD filter is to use a higher-order Lagrange interpolator that is realized as a modified Farrow structure proposed by Välimäki (1995). Also in such structures it is feasible also to modulate the delay parameter in a continuous manner with the input signal. Figure 2(b) shows the modified Farrow structure of second-order Lagrange interpolation where the mapping function G consists of a unit delay and a scaling factor c which limits the range of the time-varying delay parameter $d(n)$. The second-order Lagrange interpolator requires one multiplication and three additions more that the first-order filter and also two right shifts (multiplications by ½). Since no analytical results exist on the stability conditions of a feedback loop that includes this nonlinearity, the range of $d(n)$ must be determined experimentally. Note that in sound synthesis, input signal $x(n)$ may be a well-known sequence of samples and the range of values of $d(n)$ that will be used can thus be determined in advance.

The following example shows that the TVFD FIR filters may be used to obtain qualitatively similar effects as the allpass filter. We have experimented with a simple string model with a delay-line length of 99 unit samples and a one-pole lowpass loop filter with transfer function $b/(1 - a_1z^{-1})$ where $b = 0.9780$ and $a_1 = 0.0020$. The input signal is a synthetic pluck sound composed of two raised-cosine functions which has been integrated to attenuate the high-frequency content. A more realistic plucked-tone synthesis can be obtained by using inverse-filtered recordings as input (Välimäki *et al.* 1996). The input signal has been further processed with a feedforward comb filter that has a delay of 33 samples to cancel every third harmonic from the output signal. Figure 3 shows the magnitude response (absolute value of the FFT computed from a signal windowed 2000 samples after the attack using a 1024-point Hamming window) of synthetic tones in 4 cases: a) no nonlinearities, b) an allpass filter (coefficient switched between 0 and –0.0244, which corresponds to a fractional delay of 0.05 samples near the zero frequency), c) a first-order Lagrange interpolator, and d) a second-order Lagrange interpolator. In the two latter cases, $d(n)$ has been switched at the zero crossings so that $d(n) = 0$ when $x(n) \leq 0$ and $d(n) = 0.05$ when $x(n) > 0$. The missing harmonics have been generated in all cases. Although the results of Figure 3(c) and 3(d) are not identical with Figure 3(b), they are similar.

It is interesting to notice that the continuous delay modulation is equivalent to phase modulation, which on the other hand is equivalent to a common formulation of FM synthesis. Hence, signal-dependent FD filtering is another manifestation of frequency modulation at audio rate in digital sound synthesis!

3.2 Efficient Simulation of the Nonlinear Vibration of a String

In the following we formulate a computationally efficient digital waveguide model with uniformly distributed time-varying propagation speed. This model creates variations to the harmonic frequencies and the pitch of synthetic string sounds and generates missing harmonics. Our formulation is similar to commutation of losses and dispersion effects described by Smith (1992). In the one-dimensional digital waveguide simulation, waves $y^+(n)$ and $y^-(n)$ travel to the right and to the left in the upper and the lower delay line, respectively (Smith 1992). The temporal and spatial sampling intervals T and X of the digital waveguide are related by $c_0 = X/T$.

We assume that the time-varying propagation speed $c(n)$ is uniform along the waveguide and that it varies relatively slowly so that between time instances n and $n+1$ the wave travels a distance $\hat{X} = c(n)T$. This implies that we

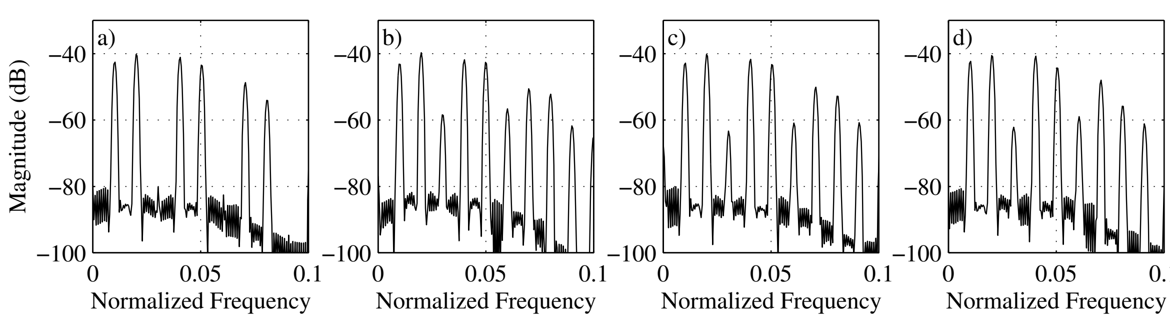

Figure 3: The magnitude spectrum of the synthetic tones produced a) without nonlinearities, b) with a switching allpass filter, c) with a switching linear interpolator, and d) with a switching second-order Lagrange interpolator.

have to resample the content of the delay lines at each sampling period. While the resampling may be performed using FD filtering, it is computationally expensive and bound to degrade the signal since an error is associated with every interpolation operation (Laakso *et al.* 1996). A more efficient implementation strategy can be utilized if we only wish to observe the traveling wave at a single or at a few spatial positions. For reasons of simplicity, in the following we only consider the wave traveling to the right. It is straightforward to treat the left-going wave similarly. Given an initial distribution of the wave variable $y(0, m)$, $m = -\infty, ..., +\infty$, the output at an observation point k at $n = 1$ is given as $y(1, k) = y(0, k - c(0)T)$, and at $n = 2$ as $y(2, k) = y(0, k - c(0)T - c(1)T)$, and so on. More generally,

$$y(n,k) = y\left(0, k - T\sum_{l=0}^{n-1} c(l)\right) \tag{1}$$

Let us express the time-varying speed as $c(n) = c_0 + c_\mathrm{d}(n)$, where $c_\mathrm{d}(n)$ is the deviation from the nominal speed c_0. Eq. (1) can be written as

$$y(n,k) = y\left(0, k - nc_0T - T\sum_{l=0}^{n-1} c_\mathrm{d}(l)\right) \tag{2}$$

In Eq. (2) $nc_0T = nX$, i.e., the distance a wave travels with speed c_0 in time n. In general, we can compute $y(n, k)$ as

$$y(n,k) = y\left(n-1, k-1 - T\sum_{l=0}^{n-1} c_\mathrm{d}(l)\right) = y\big(n-1, k-1+d(n)\big) \tag{3}$$

where we have implicitly defined $d(n)$. This implies that it suffices to utilize a single fractional delay filter operating in the vicinity of position k and approximating the real-valued delay $d(n)$. The delay term $d(n)$ depends on the time history of the deviation term $c_\mathrm{d}(n)$. This can be interpreted as compromising the locality in time for the locality in position. When resampling is used, displacement $c(n)T$ depends only on the current speed value $c(n)$. By applying the preceding treatment to the left-going traveling wave, we can generalize the above result for the bi-directional digital waveguide. After a straightforward computation we observe the bi-directional waveguide at position k as a sum of the two traveling waves expressed as $y(n, k) = y^+(n - 1, k - 1 + d(n)) + y^-(n - 1, k + 1 - d(n))$.

For practical implementations, it is advantageous to define the nominal speed c_0 so that $\langle c(n)\rangle = c_0$, where $\langle\cdot\rangle$ denotes the time average operator. This implies that $\langle c_\mathrm{d}(n)\rangle = 0$, i.e., the real-valued delay $d(n)$ oscillates around zero. Were this not the case, the actual observation positions of the two waveguides would drift apart.

4 Conclusion

In this paper we have interpreted signal-dependent nonlinearities as time-varying fractional delay filters. The applications of such techniques in physical modeling synthesis were reviewed, FIR structures for realizing nearly passive nonlinearities were introduced, and a new method to simulate a variable string tension was proposed.

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